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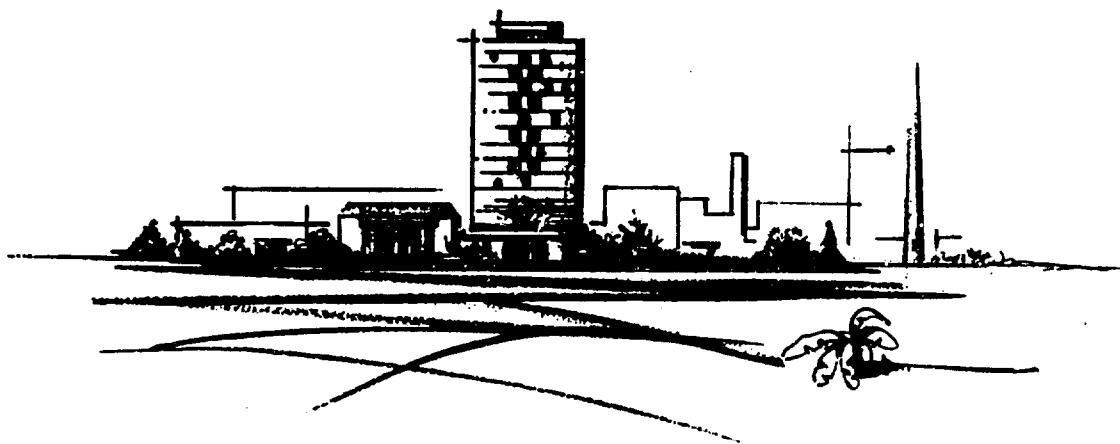
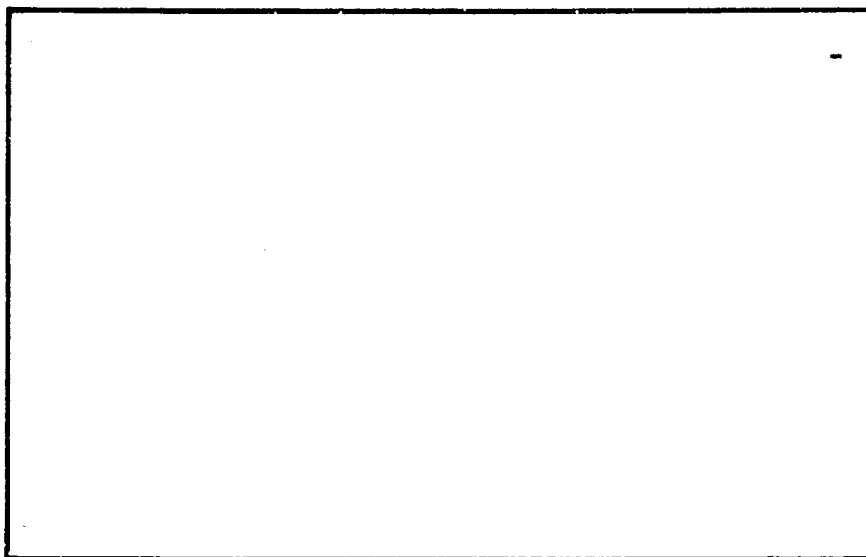
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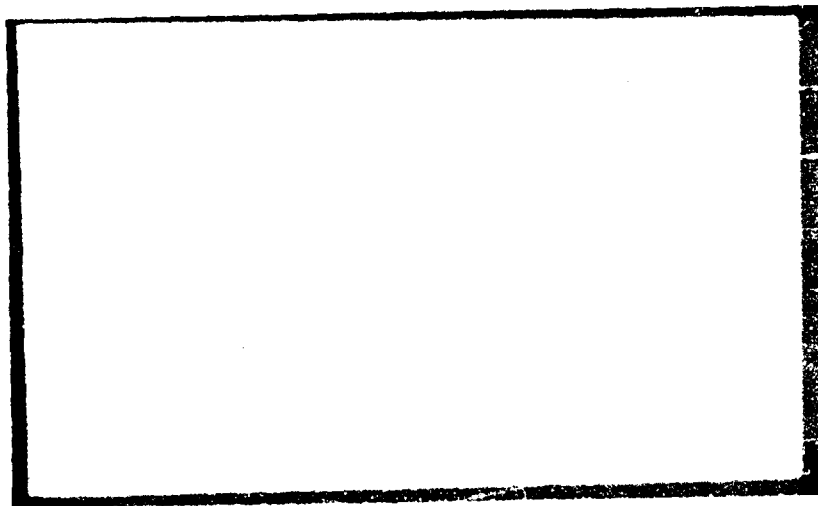


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THEORY OF GROUND-WAVE PROPAGATION ACROSS  
A ROUGH SEA AT DEKAMETER WAVELENGTHS

Donald E. Barrick

January 1970

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THEORY OF GROUND-WAVE PROPAGATION ACROSS  
A ROUGH SEA AT DEKAMETER WAVELENGTHS

by

Donald E. Barrick

ABSTRACT

The effect of sea state on ground-wave propagation across the ocean is computed in the HF and VHF region. The history and present understanding of ground-wave propagation is briefly reviewed, especially as concerns the influence of roughness. The approach of the analysis here is to derive an effective surface impedance at grazing which includes the effects of roughness. To do this, the statistical boundary perturbation approach of Rice is applied to the sea surface, which is "slightly rough" at HF/VHF. In addition, the Leontovich (or impedance) boundary condition is employed because ocean water is a good (but not perfect) conductor at these frequencies. The analysis shows that the total effective impedance at the surface can be expressed as two terms: (i) the impedance of a perfectly smooth sea water surface at grazing, and (ii) a second term accounting for roughness. The latter is obtained from the ocean wave-height spectrum.

The report examines two height-spectrum models for wind-driven ocean waves: a directional Neumann-Pierson model and an isotropic Phillips spectrum. The effective surface impedance is calculated for these models. This impedance is then used to compute the ground-wave transmission loss across the sea. Graphs are shown for a variety of frequencies, ranges, sea states, and receiver heights. Examples, in which these curves are used in communications problems are solved.

A bibliography of ground-wave (open literature) publications is included as an Appendix. The articles are arranged therein chronologically, by decades, and then alphabetically, by author.

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## I. INTRODUCTION

Interest in ground-wave propagation began with Sommerfeld's<sup>[1]</sup> analysis in 1909. Widespread utilization of these results, however, came with Norton's papers<sup>[2-4]</sup>, which reduced Sommerfeld's complex expressions to graphs suitable for engineering applications and extended the results to spherical surfaces<sup>[5]</sup>. Since then, many investigators have made contributions which include a variety of deterministic surface geometries, including layers, surface-step discontinuities, propagation beyond hills (of given geometrical shape), knife-edge discontinuities, a layered atmosphere, and abrupt changes in surface properties (i.e., a shoreline model). For several recent reviews of the pertinent literature, one should consult the articles by Wait<sup>[6]</sup>, Feinberg<sup>[7]</sup>, Bremmer<sup>[8]</sup>, Goubau<sup>[9]</sup>, King<sup>[10]</sup>, and Wait<sup>[11]</sup>. \*

With few exceptions, all of the articles on ground-wave propagation have treated the earth surface as smooth, either planar or spherical. Techniques have been developed for treating ground-wave diffraction by deterministic shapes in given locations, such as a parabolic-shaped hill<sup>[13]</sup>, a knife-edge<sup>[11]</sup>, a surface step<sup>[11,14]</sup>, or a linear interface between two surface media (modeling a coastline)<sup>[7, 15-17]</sup>. All of these obstacles, however, are handled deterministically, i.e., their shape and locations are specified, and the exact expression is sought for their diffraction effect. None of these surface irregularities, either singly or in combination, can give a really satisfying model for a statistically rough surface, such as the ocean. Hence, persons interested in ground-wave propagation across the sea have all used Norton's (or similar) techniques and assumed the sea surface to be perfectly smooth.

Yet it has been known for years that a slight corrugation on an otherwise perfectly conducting surface has a substantial effect on a wave propagating across it<sup>[18]</sup>; in particular, the reactive portion of the surface impedance is enhanced. Thus there is good reason to believe that in the HF and VHF regions, where ocean-wave heights can be a significant percent of a wavelength, the roughness will play an important role and must not be neglected. With a lack of any theoretical guidelines, measurements carried out in the past<sup>[19]</sup> have proved to be relatively meaningless, especially since

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\*A novice to the subject would do well to consult one of these reviews, with possibly a preliminary perusal of a sound tutorial treatment, such as is contained in Jordan's textbook<sup>[12]</sup> (Chapter 16). This suggestion is made because hundreds of articles have been published on the subject, and any attempt to be comprehensive is bound to be quite voluminous. Rather, the reader may refer to the Appendix for a chronological bibliography on most of the important contributions on ground waves.

no attempts were made to ascertain the statistics of the surface (i.e., the sea state) at the time of the measurements. Since an ever-increasing number of systems (both communication and radar) will involve propagation over the sea, an effort to treat the effect of surface roughness in a quantitative manner appears in order.

There have been a handful of attempts to deal with roughness in ground-wave propagation quantitatively. Feinberg<sup>[20]</sup> in an English article showed already in 1944 that roughness produces an increase in the effective surface impedance, and derived an integral for this average impedance. He obtained this result from the integral equation for the fields obtained using Green's theorem; in this respect, Feinberg's approach to the problem is similar to others who have used integral equations, e.g., Hufford<sup>[21]</sup> and King<sup>[10]</sup>. While Feinberg initially applied the Leontovich boundary condition, he later presents the effective surface impedance term contributed by the roughness; the latter contains no dependence upon the surface material properties, i.e., it is strictly valid only in the limit of a perfectly conducting surface. This useful relationship, nonetheless, apparently went unnoticed and unapplied in the West, and even in the Soviet literature it was only repeated a couple of times<sup>[22]</sup>. To the best of our knowledge, no one in either country has ever used it to study propagation across a rough sea. Bremmer<sup>[8]</sup> later notes Feinberg's result, and repeats it.

Rice, on the other hand, studied very thoroughly the problem of reflection of electromagnetic waves from a slightly rough\* surface<sup>[23]</sup>. Using the boundary conditions along with Maxwell's equations, he solves the problem using a classical perturbation approach. In his Section 6, he briefly considers propagation over a perfectly conducting rough surface for vertical polarization. His result is identical to Feinberg's if one makes the connection between his effective horizontal propagation constant and Feinberg's surface impedance. While Rice's classical paper has been quoted and applied often for scatter problems, this important result for surface-wave propagation has also gone unnoticed.

Wait<sup>[24]</sup> considers propagation along a perfectly conducting surface having a random distribution of hemispheric bosses superposed. These bosses, all of the same size, are small compared to a wavelength but their spacing is large compared to their radius; the work is based upon multiple scattering techniques derived by Twersky<sup>[25]</sup>.

\*"Slightly rough" is the accepted term for surfaces whose roughness height is less than a wavelength. The sea at HF and much of VHF falls into this category.

The interpretation of these results by Wait constitutes possibly the first attempt to consider the effect of a random roughness on the propagation of surface waves. It is obvious, however, that quantitative comparison of such a model with the sea is not promising. Senior<sup>[26]</sup> analyzes the influence of a slight roughness on the surface impedance, but unfortunately he does not consider in sufficient detail the pathological case here of grazing incidence; his results are mainly applicable at higher angles of incidence.

## II. DEFINITIONS AND BACKGROUND

Before proceeding further, it is necessary to define terms to be used here, including ground wave and surface wave. The definition employed here is that of Norton<sup>[2-5,12]</sup>. The ground wave is the total field observed at a point in space due to a radiating source a finite distance away, excluding any component reflected from the ionosphere or other discontinuities in the upper atmosphere; these latter components are termed sky waves and will not be treated here. The ground wave is then broken down further into a space wave and a surface wave. The space wave, if it exists, consists of the direct ray and that reflected from the earth; these are predicted by standard ray optics and exist only when the observation point is above the horizon. The surface wave is then the remaining field in the ground wave after the space wave has been subtracted off. Persons using this definition have alternately referred to the latter as the "Norton surface wave".

It is useful at this point to review very briefly the controversy over the definition of surface waves. While we realize that the reader of this report is little interested in a blow-by-blow description of the quarrel, it is nonetheless necessary because the analysis that follows employs some of the concepts subscribed to by both camps. The controversy involves more than a mere definition or terminology; it is a question of the nature of the fields at the surface themselves.

Sommerfeld's solution is based upon a representation of the radiation from a dipole by a summation (i.e., an integral) of plane waves. This integral, along with similar representations for the reflected and transmitted fields, satisfy Maxwell's equations and the boundary conditions at the surface. The resulting expression for the total field above the surface is then expanded asymptotically (valid where distance from

the source is large in terms of wavelength). When the terms appearing in this asymptotic representation are interpreted physically, the "space wave" terms described above stand out; hence, it was natural to call the remainder of the solution a "surface wave". As one approaches grazing propagation on a smooth planar surface of finite conductivity, the space wave vanishes as it should due to the cancellation of the direct and reflected waves; thus any power received is due to propagation via this "surface wave". Here, however, one can encounter difficulties; one at first might expect that this "surface wave" alone should satisfy Maxwell's equations and the boundary conditions; it does not. This fact has caused doubt in the minds of some, who felt that the entire expansion was somehow erroneous or inadequate. It should have been no particular cause for alarm, however. Many popular asymptotic expansions of scattered and diffracted fields fail these requirements when individual terms are interpreted separately. Physical optics fails these conditions in certain regions of space, and yet it proves to be a valuable tool when used properly. Individual creeping wave terms in Keller's geometrical theory of diffraction<sup>[27]</sup> fail these requirements, yet this asymptotic technique is considered an important and meaningful approximation. The point to be made here is that it is not unusual that separate terms of a high-frequency asymptotic expansion will fail to satisfy Maxwell's equations and the boundary conditions, even though certain of these terms appear to be separate physical entities capable of standing alone. When one employs all of the terms in the asymptotic expansion together, and approaches the asymptotic limit (usually allowing frequency to become infinite), the entire result will nearly always satisfy these requirements.

Zenneck<sup>[28]</sup> first called attention to the fact that when one merely solves Maxwell's equations at a planar interface between two homogeneous media (one of which may be slightly dissipative, representing the ground), and applies the boundary conditions, one arrives at a field which appears to be "attached" to the surface (i.e., its amplitude attenuates exponentially with height) and which falls off exponentially along the propagation direction. This solution satisfies the needed requirements, but assumes the media are source-free.\* The exponential attenuation in the propagation direction is not entirely unfamiliar, and explains the removal of energy from the field; in this sense it complies with conservation of energy. This removal of energy occurs here due to the dissipation of the ground, which converts electromagnetic energy to heat. In other cases, such as propagation across a

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\*In this respect the approach is similar to "assuming" incident plane wave fields everywhere, which is commonly done in certain situations.

slightly rough surface or propagation through rain, this attenuation is due to removal and scatter of constant fractional increments of energy per unit length along the propagation direction. Since this wave obviously satisfied the proper equations to justify its existence at an interface, Zenneck felt that it should be called a "surface wave". Unfortunately, an error in Sommerfeld's original analysis resulted in a form for Sommerfeld's surface wave which coincided with Zenneck's definition. Zenneck took this as convincing proof for his explanation of radio wave propagation over a surface. Sommerfeld corrected this oversight in a 1926 work<sup>[29]</sup>, which was apparently not noticed by everyone. By 1936, serious questions began to arise concerning the importance of Zenneck's wave with radiation from typical, finite-length, radio antennas; this attention was spurred by experimental evidence which showed propagation behavior unpredicted by the Zenneck wave. Several investigations at that time by Norton<sup>[2]</sup>, Rice<sup>[30]</sup>, and Burrows<sup>[32]</sup> found the error in Sommerfeld's original work and obtained asymptotic expansions for the radiated field exhibiting a common behavior. All of these works, along with Sommerfeld's 1926 paper, showed that if the Zenneck wave was present, it certainly would not be dominant at large distances from the source. Yet they did show<sup>[31]</sup> that the ratio between the horizontal and vertical components of the electric field at the surface (i.e., the wave tilt) is nearly identical to that for the Zenneck wave. This ratio is proportional to the normalized surface impedance near grazing incidence, and hence the definition and use of surface impedance for the description of ground-wave propagation is valid. Wait<sup>[34]</sup> confirms this viewpoint and gives a summary of ground-wave propagation from a vertical dipole based upon the properties of the surface impedance. Furthermore, Norton<sup>[4]</sup> showed that the Poynting vector direction for his surface-wave component coincides nearly perfectly with that predicted for the Zenneck wave when the surface impedance is relatively low. This Poynting vector, being directed slightly into the surface, indicates that the energy is flowing into the lower medium at an angle very close to the Brewster (or "pseudo-Brewster") angle.



Confusion and controversy continued.\* Several other treatments of radiation from a finite source were developed based upon integral equations, a different approach from that used by Sommerfeld and Norton. Feinberg<sup>[20]</sup> and Hufford<sup>[21]</sup> employed the integral equation resulting from the application of Green's theorem to the Helmholtz equation (the latter arising from Maxwell's equations). King<sup>[10]</sup> used a different integral equation originating from the "Compensation Theorem". All of these analyses are based upon radiation from a finite source, e.g., a dipole, above a planar earth; such a source in the absence of the plane would radiate a spherical wave. They all arrive at the same asymptotic equation as the Norton-Sommerfeld result. Over the years, experimental evidence for radio-wave propagation has tended to confirm these asymptotic expressions, rather than the rapidly attenuating Zenneck-wave explanation. The charge that the asymptotic expansion for the Norton surface wave does not satisfy Maxwell's equations and the boundary conditions is answered in a recent paper by King<sup>[10]</sup>. He shows that the required correction is arbitrarily small if the source and observer points are close to the surface and the surface impedance is quite low (the latter condition implying an electrically dense and/or lossy lower medium). These conditions are nearly always satisfied in the case of propagation over the earth.

Based on the concurrence of the several analyses and experimental evidence, one must conclude that the Norton-Sommerfeld result properly explains propagation over a plane<sup>†</sup> earth when the field radiates from a finite-sized source (e.g., a dipole). The Zenneck wave is not postulated on radiation from a finite-sized source, and hence

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\*The need for some sort of "meeting of the minds" reached a culmination in the late 1950's, when a working group was set up by U.R.S.I. (International Scientific Radio Union) under the chairmanship of J. R. Wait to agree upon suitable definitions on the nature of surface waves. This group more or less dissolved at the URSI 13th General Assembly in London, England, in 1960 when it appeared that further discussion on the topic at that time would not be fruitful. The Proceedings of that meeting contain discussion by participants reflecting their views on the subject (pp 533-539); more data on the subject by many of the principal participants can be found in the Transactions of the IRE on Antennas and Propagation, Vol AP-7, Special Supplement, pp 132-243A (1959). Wait<sup>[34]</sup> later provided a needed comprehensive historical review of the subject; this is both objective and quite thorough.

†The nature of the surface-wave over a spherical earth is fraught with the same general problems, i.e., does a wave of the Zenneck type dominate? Only the plane-earth case is reviewed here for brevity.

it does not apply in the ground-wave propagation problem since the source-generated asymptotic expansions show no evidence of its existence. Nobody has yet designed an antenna which can excite a Zenneck wave over a plane earth (if such a finite-sized antenna is even possible), although Barlow and Brown<sup>[18]</sup> discuss this subject. Therefore, one must employ the source-postulated approach, along with the asymptotic expansions attributed to Norton and Sommerfeld (both for the plane and spherical earth) when analyzing ground-wave propagation over the earth.

Nonetheless, paradoxes remain. If the Zenneck wave is not an important component in the groundwave from a source, why does the polarization or wave tilt obtained from the asymptotic solution agree so closely with that of the Zenneck surface-wave? Why does the energy at the surface propagate into the ground at the same angle, i.e., the Brewster angle, for both cases? Are these facts merely a coincidence? Wait<sup>[34]</sup> points out that rather than coincidence, this near-equivalence of the wave tilt at the surface is a consequence of the fact that the Zenneck-wave pole in the Sommerfeld integral is near the free-space wavenumber in the complex wavenumber plane. This mathematical connection between the two, however, does not provide a clear physical interpretation of their relationship. An exact solution of Maxwell's equations and the boundary conditions, in the absence of sources, shows that such a solution must be a Zenneck wave, which is nearly planar and attenuating in both the height and propagation directions, over a restricted localized region near the surface<sup>[12,18]</sup>. Why is such behavior not present at great distances from the source in the asymptotic results of Norton? Needless to say, until these points are answered in physically meaningful manner, controversy and confusion is bound to continue.

### III. APPROACH AND SCOPE OF PRESENT ANALYSIS

The purpose of the present analysis is to quantitatively establish the effect of sea state on ground-wave propagation and attenuation over the ocean. Due to the nature of the approximations made, the results will be valid at frequencies in and below the VHF region; vertical polarization only is considered.

The first portion of the analysis will derive an expression for the effective (or average) surface impedance of a roughened surface. Instead of treating the surface as a perfect conductor, we shall use the Leontovich boundary condition for

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good (but finitely) conducting surfaces; this condition is especially applicable to ocean water in the HF and VHF regions. Even though the sea surface may appear to be a good conductor, the distinction between a good conductor and a perfect one at these frequencies is an important one. A ground wave propagating across a smooth perfectly conducting surface suffers no attenuation due to the presence of the surface, and propagates as though it were in free space. A good (but not perfect) conducting smooth surface will produce attenuation of the ground wave at the surface<sup>\*</sup>; for sea water, this attenuation factor results in loss in signal level of tens of decibels at ranges of 50 miles or greater even if the sea is treated as perfectly smooth. Hence, the present analysis of roughness will take into account this finite conductivity from the outset. The Leontovich boundary condition, in contrast with other approaches, is discussed in the next section.

As shown in Senior's analysis<sup>[26]</sup>, the effective surface impedance for any surface is a function both of the angle of incidence and polarization. Since only vertical polarization is considered, the remaining question arises as to the angle of incidence to be used here. This is important, because results useful for scatter at higher incidence angles are not usually meaningful when this angle is reduced to the near-grazing region common in ground-wave propagation, and one must treat this case separately. To do this, a plane wave will be assumed to be guided by the surface, and the effective vertical wave number will be determined and related to the surface impedance. In this respect, the approach taken is the same as that in Rice<sup>[23]</sup> (i.e., a perturbation approach), except for the difference in boundary conditions. This guided wave is identical to the Zenneck wave discussed previously. One may ask why such an approach has validity here when we ultimately want to consider a finite source which produces no Zenneck wave in the radiated far field? The justification is based upon the discussion in the preceding section; there it was mentioned that results of Norton<sup>[4]</sup> and Wise<sup>[31]</sup> show that the Norton surface-wave derived from the finite source radiator, has the same phase and polarization tilt (i.e., direction of incidence) as the Zenneck wave, even though the attenuation in no way resembles that of the Zenneck wave. Hence, the effective surface impedance derived by employing a Zenneck guided wave, being applicable for the same incidence angle, can be used in the source-based formulation of Norton<sup>[2]</sup> and Wait<sup>[33,34]</sup>. The

<sup>\*</sup>This has been defined in the preceding section as the Norton surface wave.

reasons that we initially assume a plane wave propagating in a guided mode (Zenneck wave) across the surface are primarily for the mathematical convenience and the physical insight obtainable. A check on the validity is obtained by comparison of result with that of Feinberg<sup>[22]</sup> where we permit the conductivity to approach infinity. Feinberg's result was derived from an integral equation approach involving the source. Besides providing an alternate derivation from Feinberg's, the analysis here shows explicitly the effect of the finite conductivity in the roughness contribution to surface impedance. This addition indicates the limits of validity in Feinberg's assumption, and also yields a correction factor which accounts for this deviation from perfect conductivity. Furthermore, it obviates a numerical difficulty encountered in computing the integral for a given sea surface model.

With the above technique, we derive an expression for the average, or effective, surface impedance. This is the sum of two terms: the first being the normal impedance of the otherwise smooth surface, and the second term representing the additional effect of the roughness. The scale and spatial height spectrum of the roughness (or the roughness wavelengths) determine the nature of this impedance change. In the HF/VHF region, the radio wavelengths are of the same order as the typical ocean wavelengths, and a noticeable effect on the impedance is expected. In order to study this in a more quantitative manner, we select two semi-empirical models for the sea surface: the Neumann-Pierson spectrum and the Phillips spectrum. In the first model, the ocean waves are assumed to be slightly directional in nature, while in the latter, a non-directional (or isotropic) angular dependence is assumed. These models express the ocean wave-height strength in terms of wind speed, assuming that this wind has been blowing sufficiently long that the resulting sea state is fully developed<sup>[35]</sup>. Using the models, values of effective surface impedance are computed numerically. This is done for a variety of frequencies from 1 to 500 MHz, and for five sea states.

The reason for computing the effective impedance of the rough sea is to eventually calculate the strength of the field in the ground wave radiated from a source above such a surface. This is done using the computer program of Berry and Chrisman<sup>[36]</sup> for ground-wave propagation over a spherical earth for dipole antennas. In other words, after the average surface impedance is calculated which includes the roughness effects, we assume that the ground consists of a smooth surface with this equivalent impedance. This procedure is in the spirit of Wait's article<sup>[33]</sup> which

suggests this, for example, for corrugated surfaces. Curves plotted from this program are included which show the basic transmission loss to various points above the earth as a function of sea state, frequency, range, and height. This is done for propagation in both the downwind and crosswind directions so that some variance of signal strength can be obtained.

Another effect occurs as a result of the roughness. The ground wave propagating across the sea interacts with the ocean waves, and a portion of the energy in it is removed and scattered off into the sky. This scatter does not take place for smooth surfaces. Hence the total field arriving at any point above the surface is made up of two components: the ground wave which would exist at this point for a perfectly smooth surface with the equivalent impedance calculated, along with the scatter from all points on the surface due to roughness\*. The former component is coherent, meaning that a CW transmitted signal will arrive at this point as a pure CW signal. The latter scatter, however, is incoherent, in that its spectrum will no longer be a pure CW sine wave, but will be modulated by the time-varying ocean surface. The spectral spreading of the latter can be of the order of several tenths of a hertz. Curves of the total average intensity of this incoherent component will be shown in a subsequent report and compared to the coherent power in the ground wave below it at the sea surface.

Additional subjects such as the sea clutter power, its spectrum, and bi-static geometries, will be deferred to a later report. A section will describe the use of the transmission loss curves for communications applications.

#### IV. ANALYSIS

##### A. Formulation of Various Boundary Conditions

##### 1. General Conditions at Interface Between Two Homogeneous Media

To solve any problem involving the interaction of an electromagnetic wave with an interface between two media, one must relate the field quantities at the

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\*This does not take into account any ionospherically-reflected components.

interface by means of constraints commonly referred to as boundary conditions. The most basic and general conditions are derived directly from Maxwell's equations in their integral form. These conditions are stated as follows: (i) the tangential components of the electric and magnetic (E and H) fields must be continuous across the interface, and (ii) the normal components of the electric and magnetic flux densities (D and B) must be continuous across the interface. This boundary condition is valid for any arbitrarily curving interface. Unfortunately, it involves explicitly the fields on both sides of the interface (except when the surface material is a perfect conductor; this will be discussed in the next section), which increases the complexity of solving many typical scatter problems.

## 2. Perfectly Conducting Surface

When the surface material is a perfect conductor, the above boundary conditions simplify to the following: (i) the total tangential electric field above the surface is zero, or in equation form,

$$\vec{E}^T - (\hat{n} \cdot \vec{E}^T)\hat{n} = 0 \quad (1)$$

and (ii) the total normal magnetic field at the surface is zero, or in equation form,

$$\hat{n} \cdot \vec{H}^T = 0 \quad (2)$$

In the above equations,  $\hat{n}$  is the unit normal to the surface,  $\vec{E}^T$  and  $\vec{H}^T$  are the total electric and magnetic fields at the surface. Again, these conditions are valid at any perfectly conducting surface, regardless of whether it is curved or planar, and independent of the angle of incidence. They are more convenient usually than those in (1) because they involve only the fields above the surface, in the medium of interest. They do, however, involve only tangential E and normal H; i.e., there are no similar, simple relations for normal E and tangential H at a perfect conductor. To obtain such relationships in terms of the latter two components, one must solve Maxwell's equations, and the results are not especially tractable or useful.

### 3. Planar Interface Between Homogeneous Media--Fresnel Reflection Coefficients

One case in which a simple boundary condition obtains for general homogeneous materials occurs where the interface is perfectly smooth and planar. In this case the total field on one side of the surface can be expressed entirely in terms of the Fresnel reflection coefficients and the incident (locally plane) wave. These reflection coefficients are functions of the incidence angle and surface material constants. Any dissipative loss in the material (either electric or magnetic) can be included in these coefficients as a complex permittivity and/or permeability. The elimination of the fields on the other side of the surface in this boundary condition is convenient, but unfortunately the entire method is strictly valid only for perfectly smooth, planar interfaces (surfaces). This is in contrast to the boundary condition for the perfectly conducting surface discussed previously, which is valid for surfaces with any degree of curvature. That boundary condition applied only to the total tangential electric field at the surface, whereas the one discussed here for planar surfaces applies to the total (both normal and tangential components) electric and magnetic fields at the surface. How to deal with curving surfaces will be considered subsequently.

### 4. Curving Interface--Tangent Plane Approximation and Fresnel Coefficients

The preceding section showed that the total fields at a perfectly planar surface can be written directly in terms of the incident field times an expression involving the Fresnel reflection coefficients. The question arises: under what circumstances can such a convenient expression be extended to curving and rough surfaces? This question has been examined in many places, and the general conclusions are that it is valid when applied to surfaces whose local radii of curvature are much greater than wavelength. In fact, the correction term in this approximation can be shown to be of the order of  $(k_0 \rho)^{-1}$ , where  $k_0 = \frac{2\pi}{\lambda}$  ( $\lambda$  is the free-space wavelength), and  $\rho$  is a radius of curvature of the surface. When wavelength and surface curvatures are such that the approximation is reasonably valid, one writes the total field at a given point on the surface in terms of the incident field and the reflection coefficients, the latter being taken at the angle of incidence to that point. Hence the

resulting expression for the total field at the surface varies from point to point with the local incidence angle; this local incidence angle is defined as the angle between the incident wave propagation direction and the normal to the surface at that point. The formulation of the total fields at the surface in such a manner is called the tangent-plane approximation; it is often used along with physical and geometrical optics methods to compute the fields reflected and scattered from curving surfaces in the high-frequency limit.

The tangent-plane approximation carries two further restrictions with it. First, at regions of the surface not directly visible to the incident wave (i.e., shadowed regions), the fields are assumed to be identically zero. This vanishing of the fields in shadowed regions is not strictly valid in itself. Furthermore, it introduces complications in having to determine (either deterministically or statistically) the regions shadowed for different incidence angles. Near grazing, this shadowing effect becomes so serious that its neglect cannot be tolerated; yet it is at near-grazing angles that surface waves appear to propagate. A second restriction inherent in the tangent-plane approximation is the lack of multiple scattering between two or more surface points. This effect is neglected because the total fields at each surface point are written in terms of the incident field only at that point, which excludes any field contribution reflected from a nearby surface point. Multiple scattering is expected to be a serious contributor only for surfaces having many large concave regions, and is not typical of the sea surface, for example.

#### 5. Impedance (Leontovich) Boundary Condition-- Curving Surface

A further boundary condition which can be very useful for certain situations is called an impedance boundary condition. In particular, when applied to vectorial electromagnetic waves, it is commonly referred to as the Leontovich boundary condition because of the detailed pioneering investigations of Leontovich<sup>[37,38]</sup> on this subject in the 1930's. He showed that when the conductivity and refractive index of the medium below the interface are relatively large, one can arrive at an expression for the total tangential fields immediately above the surface which does not involve fields in the medium at all. Furthermore, they do not require serious restrictions on the surface radii of curvature, as does the tangent-plane approximation discussed in the preceding section.



In equation form, the Leontovich boundary condition appears to be a slight extension of Equation (1) for the perfectly conducting surface:

$$\vec{E}^T - (\hat{n} \cdot \vec{E}^T)\hat{n} = Z_s(\hat{n} \times \vec{H}^T) \quad (3)$$

Here,  $\vec{E}^T$  and  $\vec{H}^T$  are the total electric and magnetic fields above the surface and  $\hat{n}$  is the unit normal to this surface. The quantity  $Z_s$  is termed the surface impedance. For a homogeneous medium below the surface,  $Z_s$  is given by

$$Z_s = \sqrt{\frac{\mu}{\epsilon}} \quad (4)$$

where  $\mu$  and  $\epsilon$  are the absolute permeability and permittivity of the material.

Obviously, the above boundary condition carries certain restrictions on the media and interfaces to which it applies. As derived by Leontovich, these restrictions are the following: (i) the index of refraction (i.e.,  $\sqrt{\mu\epsilon}$ ) of the material is large and has a large imaginary part, (ii) the fields immediately above the surface vary slowly along the surface over a distance of the order of a wavelength in the material (i.e., a wavelength in the material is  $\lambda_m \approx \left|\frac{1}{\sqrt{\epsilon\mu}}\right|\lambda_0$ , where  $\lambda_0$  is the wavelength in free space), and (iii) the radii of curvature of the surface are small compared to  $\lambda_m$ , the wavelength in the material.

Thus, while there is a restriction on the radius of curvature of the surface, it is considerably less stringent than that for the tangent-plane approximation. The condition there was that the radii of curvature had to be large compared to  $\lambda_0$ , the wavelength in free space. Leontovich<sup>[38]</sup> in fact derives the first-order correction terms for the above boundary condition (Equation (3)) and finds that for a homogeneous material, they contribute terms to the right side of (3) of the order of

$$Z_s(\hat{n} \times \vec{H}^T) \left[ \frac{2}{ik_0\sqrt{\mu\epsilon}} \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \right] \quad (5)$$

where  $\rho_1$  and  $\rho_2$  are the principal radii of curvature of the surface at any point.

Investigators have found that the impedance boundary condition expressed in (3) is useful not only for homogeneous media below the surface, but also for other types of surfaces such as corrugated and dielectric-clad conducting surfaces. In

fact, the purpose of this report is to derive an effective  $Z_s$  for a randomly rough sea surface. A rather thorough review of the Leontovich boundary condition and the general impedance boundary condition for various types of surface conditions was done by Senior<sup>[39,26]</sup>.

#### B. Applicability of Boundary Conditions to the Sea at HF/VHF

Not all of the above boundary conditions are applicable to a rough sea in the HF/VHF region. The boundary condition selected depends upon the method of attack used to solve the problem and upon the material and surface properties of the ocean. Several investigators have employed questionable (and even inapplicable) boundary conditions and techniques in the past to study sea scatter at HF/VHF, and this is the reason that the question is studied rather thoroughly here. It is the conclusion of this section that the only strictly valid technique to determine scatter from, and propagation along, a rough sea at HF/VHF is the boundary perturbation approach attributed to Rice<sup>[23]</sup>. Using this technique, the only strictly applicable boundary condition is the Leontovich condition, for scatter and propagation at near-grazing incidence.

#### 1. General Conditions at Interface Between Two Homogeneous Media

Rice initially formulated a solution for scatter from a slightly rough random surface between two dielectric media for a horizontally-polarized incident wave. His results were extended to vertical incident polarization by Peake<sup>[40]</sup>, and by Barrick and Peake<sup>[41,42]</sup> to homogeneous surfaces with arbitrary  $\mu$  as well as  $\epsilon$ . All of these analyses are based upon a perturbation expansion of both the surface and the reflected fields. The boundary condition used is the general condition involving the fields on both sides of the interface, as discussed under 1. of the preceding section. While the expansion is messy algebraically, it is straightforward, and results may be found in the above references.

As applied to the sea surface at HF/VHF, these results are believed to be very questionable, if not entirely invalid. This is due to the fact that the expansion of the perturbed fields beneath the surface is based upon a simple series representation of the quantity  $e^{ic(m,n)\zeta}$ , in which only first three terms are

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retained. This is valid if  $c(m,n)\zeta$  is small ( $\zeta$  is the ocean wave height above a mean level). However,  $c(m,n)$  is defined as

$$c(m,n) = k_0 \sqrt{\epsilon_r \mu_r - \frac{a^2 m^2}{k_0^2} - \frac{a^2 n^2}{k_0^2}} \approx k_0 \sqrt{\epsilon_r \mu_r - \sin^2 \theta_s} \quad (6)$$

where  $\theta_s$  is the scattering angle from the vertical to the earth, and  $\epsilon_r, \mu_r$  are the relative constitutive parameters, which in general may be complex. For the sea the dielectric constant is about 80 and the average conductivity is about 4 mhos/m. At 10 MHz, this gives  $\mu_r = 1$ ,  $\epsilon_r = 80 + 17200j$ . For real scattering angles  $\theta_s$ ,  $|c(m,n)|$  is of the order of  $85 k_0 \approx 18 \text{ m}^{-1}$ . Hence, even for a very calm sea with ocean waves whose heights,  $\zeta$ , are of the order of  $6 \text{ in}$ , the quantity  $|c(m,n)\zeta|$  is of the order of 3, and clearly, the first three terms of the exponential  $e^{ic(m,n)\zeta}$  are entirely insufficient. In addition, the representation of the solution with a perturbation formulation, where  $c(m,n)\zeta$  is a "smallness" parameter, is invalid because such a smallness parameter must always be considerably less than unity.\* The above restriction on ocean waves fails throughout the entire HF/VHF region.

## 2. Perfectly Conducting Surface

The ocean at HF/VHF is a good conductor, and for many applications may be considered a perfect conductor. For our application here, however, such a simplification is not possible. The following is an explanation of the reason. A ground wave propagates in a direction close to grazing, but actually appears to be propagating into the surface at the Brewster angle. For sea water, this angle at 10 MHz is about  $1/2$  degree from grazing. For the vertical polarization states, propagation across a perfectly smooth, perfectly conducting surface takes place with no attenuation (other than that of free space). In addition, scatter from a perfectly conducting, slightly rough surface is entirely insensitive to grazing angle in this region, i.e., the scattered power for incidence and scattering angles of about  $2^\circ$ ,  $1/2^\circ$ , and  $0^\circ$  above grazing are identical.

\*The discussion here is not meant to imply that Rice's analysis using the interface boundary conditions between two homogeneous media is invalid in general. There are a great many surfaces and materials which meet this restriction. The condition is easier to satisfy for less dense surface materials; sea water, however, is quite dense electromagnetically at HF/VHF.

For a good, but not perfect, conductor however, propagation near grazing in the vertical polarization states differs considerably from the behavior described above. First, ground-wave propagation across a perfectly smooth sea surface will suffer several tens of decibels attenuation over that a perfectly conducting surface at a range of, say, 100 miles. Hence, by assuming a perfectly conducting sea, one loses entirely the ground-wave attenuation function which is the essence of the problem. Second, when such a surface is slightly rough, the scatter for the vertical polarization states has been shown to vary considerably at angles near grazing<sup>[42]</sup>, in contrast with the case of a perfectly conducting surface.\* Here again, the difference between a good and a perfect conductor is critical.

For HF/VHF, and for most sea scatter problems involving horizontal and vertical polarization (where incidence and scatter angles are restricted from the region within 3° of grazing), the assumption of a perfectly conducting surface is valid. In these cases, the Rice perturbation technique based upon equation (1) is entirely adequate, and results derived and presented for average scattering cross sections in<sup>[40-42]</sup> are applicable (i.e., where a perfectly conducting surface is assumed). For the near-grazing angles associated with ground-wave propagation, however, the assumption of perfect conductivity can lead to dangerous and possibly erroneous results and interpretations, as seen from the discussion in the preceding paragraph.

### 3. The Tangent-Plane Approximation

The tangent-plane approximation, as discussed in the preceding section, replaces the total field at the surface by the incident field times a factor involving the Fresnel reflection coefficients. These coefficients are strictly valid only at a perfectly smooth, planar interface, but may be employed with little error when the radii of curvature of the surface at a given point are much larger than wavelength. When this condition is met, the reflected field at the given point is treated as though it had been reflected from an infinite plane tangent to the surface at that point: hence the term "tangent-plane" approximation. The approximation is commonly used to reduce an integral equation for the fields over the surface to a definite integral, the latter being called the "physical optics" integral.

\*The subject of incoherent scatter from a ground wave will be discussed in a separate report, and this contrast in behavior between good and perfectly conducting surfaces near grazing will be examined.

In order to be applicable for rough surface scatter problems, the radii of curvature of the surface at most points must be considerably greater than a wavelength. For the ocean, the dominant sea waves which will contribute to HF/VHF scatter are those of the order of one-half the radio wavelength. To check the applicability, assume a frequency of 10 MHz, and hence, a free space wavelength of 30 meters at winds of about 10 knots, 15 meter-long ocean waves will be excited, and their heights will be of the order of  $\zeta = 2$  to 3 feet. Such waves (assume they are nearly sinusoidal) will have a radius of curvature at the peak and trough of about 7 meters, considerably less than the radar wavelength. Thus, the restriction on the radius of curvature at HF/VHF fails, and the use of the tangent-plane approximation here is extremely questionable.

A further argument against the use of the tangent-plane approximation with the physical optics integral for propagation near grazing is the inherent neglect of shadowing in the approximation. Shadowing at these angles will be a very serious factor, and its neglect near grazing seems an oversimplification.

Various investigators have nonetheless employed this approximation and tried to extend its use of HF/VHF scatter from the sea. As evidence of the inconsistencies resulting from its use, no polarization dependence is predicted for scatter within the plane of incidence. Yet measurements show considerably stronger scatter for the vertical polarization states than for the horizontal states. For ground-wave propagation, polarization is of the essence of the problem, and any theory which shows no polarization dependence cannot be used. Therefore, it is concluded that the tangent-plane approximation cannot be used here for ground-wave propagation, interaction, and scatter from the sea in the HF/VHF region.

#### 4. Leontovich Boundary Condition

The Leontovich (or impedance) boundary condition is applicable to surfaces whose radius of curvature is larger than the radio wavelength inside the materials as discussed in a previous section. For the ocean this wavelength is  $\lambda_s = \lambda_0 / |\sqrt{\epsilon_r}|$ , where  $\epsilon_r$  is the complex dielectric constant of sea water. At 10 MHz, this becomes  $\lambda_s \approx 1$  foot. As mentioned in the preceding section, the smallest radius of curvature for the ocean waves producing scatter at these frequencies is about 7 meters or 21 feet. Hence, the requirement on the applicability of the Leontovich boundary condition is readily fulfilled for the HF/VHF region and the sea surface.

In employing this boundary condition along with the Rice boundary perturbation technique, one therefore avoids the necessity of writing the fields above the surface in terms of the fields beneath the surface. Since the technique is now based upon an expansion in  $k_0\zeta$  (i.e., the free-space radio wavenumber times the surface height) and not in terms of  $k_s\zeta$  (i.e., the wavenumber in sea water times the surface height), the difficulties discussed in Subsection 1. above are avoided. At HF/VHF, the parameter  $k_0\zeta$  is considerably less than unity (for frequencies below about 50 MHz and peak-to-trough wave heights below about 4 feet), and the perturbation expansion about such a parameter converges rapidly. In addition, shadowing is not neglected with this technique, as it is with the tangent plane approximation.

Therefore, we conclude that the use of the Leontovich boundary condition along with the Rice perturbation technique gives the soundest analysis of ground-wave propagation and scatter across a rough sea at HF/VHF.

### C. Guided Wave at Smooth Impedance Boundary

At any smooth, planar interface between two media, various types of semi-infinite guided waves can be shown to exist. They are semi-infinite in the sense that they can extend to infinity in directions normal to the interface. They represent mathematical solutions to Maxwell's equations which are forced to satisfy the boundary conditions at the interface. The existence of this type of guided wave is postulated without regard to the reality of sources which might be required to excite them. In this sense they are analogous to the concept of semi-infinite "plane waves" in free space, the latter merely satisfying Maxwell's equations without regard for the physical reality of their sources. Like plane waves, the physical picture of guided waves is believed to be valid over a small localized region near an interface far from the sources producing them. As mentioned in Chapter II, Wise<sup>[31]</sup> and Norton<sup>[4]</sup> have shown that the Sommerfeld ground wave excited by a finite source has the same polarization relationship and angle of incidence near the interface as the guided wave.

The term "guided wave" is employed here in order to avoid the use of the controversial term "surface wave". Actually, the guided waves, as we intend to use the term, is identical to the "Zenneck surface wave" often found in the literature. Two types of simple guided waves can be shown to exist at a planar interface and satisfy Maxwell's equations: plane guided waves and radial guided waves. The former

represents the solution in rectangular (or Cartesian) coordinates; this wave, like a plane wave, extends to infinity along a line parallel to the surface and perpendicular to the propagation direction. The radial wave, in contrast, represents the solution in cylindrical (or polar) coordinates; the wave fronts appear as circular, concentric rings emanating from the z-axis normal to the interface. We shall deal only with the former, or the planar guided waves because of the convenience of expanding the surface in rectangular coordinates.

Any elementary electromagnetics textbook can be consulted on the derivation and analysis of guided waves.\* The procedure is so standard that the details will not be repeated here. The technique may be summarized as follows: (i) solutions to the wave equation are found in rectangular coordinates for the media on both sides of the interface, (ii) the constants (i.e., multiplicative amplitude factors and also wavenumbers appearing in the exponentials) are then determined by applying the boundary conditions at the interface. Using this technique and restricting attention to impedance boundaries of interest in this application, the solution for the electric field components of the guided wave above the surface can be written as follows (see Figure 1):

$$E_z = E_0 \exp[ik_0 \sqrt{1 - \Delta^2} x - ik_0 \Delta z - i\omega t] , \quad (7a)$$

$$E_x = E_0 \Delta \exp[ik_0 \sqrt{1 - \Delta^2} x - ik_0 \Delta z - i\omega t] , \quad (7b)$$

$$H_y = \frac{-E_0}{120\pi} \exp[ik_0 \sqrt{1 - \Delta^2} x - ik_0 \Delta z - i\omega t] , \quad (7c)$$

where  $E_0$  = E-field amplitude constant,  $Z_s$  and  $\Delta$  are the surface impedance and normalized surface impedance respectively. The latter conforms to the notation of Wait<sup>[33,34]</sup>, and is defined as follows:

$$\Delta = \frac{Z_s}{120\pi} . \quad (8)$$

The solutions given in (7) are based on the assumption that  $|\Delta| \ll 1$  (which is true for sea water at HF/VHF). Note that as the surface properties approach a perfect conductor  $\Delta \rightarrow 0$ , and Equations (7a,b,c) degenerate to plane-wave free-space propagation along the x-direction.

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\*Jordan's text<sup>[12]</sup> (Section 7.09) provides an analysis of guided waves. A more detailed treatment which includes both planar and radial guided waves and various types of surfaces can be found in Barlow and Brown<sup>[18]</sup> (Section 2.1).

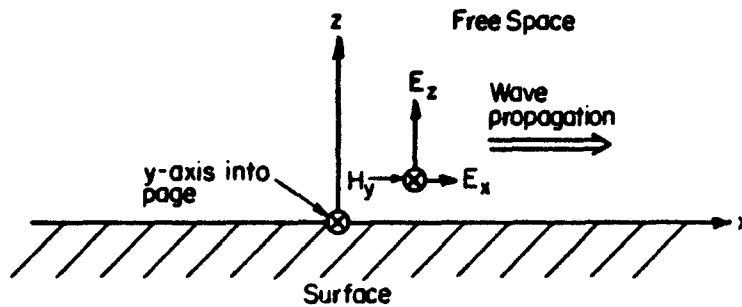


Figure 1. Guided Wave Above a Planar Impedance Boundary

For a homogeneous material below the interface,

$$\Delta \approx \sqrt{\frac{\mu_r}{\epsilon_r}}, \quad (9)$$

where  $\mu_r$  and  $\epsilon_r$  are the relative permeability and permittivity of the medium.\* Either or both may be complex. For our application, sea water has approximately the following constants:

$$\mu_r \approx 1, \quad \epsilon_r \approx 80 + i \frac{\sigma}{\epsilon_0 \omega}, \quad (10)$$

where  $\epsilon_0 = 10^{-9}/36\pi$ ,  $\omega$  is the angular frequency, and  $\sigma$  is the conductivity of sea water.<sup>†</sup> At 10 MHz, this value is approximately  $\epsilon_r = 80 + i7200$ , and thus  $\Delta \approx 1.18 \times 10^{-2} \cdot e^{-i\frac{\pi}{4}}$ . (The time dependence,  $e^{i\omega t}$ , will be dropped henceforth.)

The solutions for the guided wave represented by Equations (7a,b,c) are valid for a smooth sea surface. When the surface becomes rough or corrugated, the normalized surface impedance,  $\Delta$ , appears to change. For a deterministic corrugated surface, for example, whose period is much less than a radio wavelength, this change manifests itself as a sharp increase in the reactive (or imaginary) component of  $\Delta$ . In particular, the surface appears inductive. It is this increase in  $\Delta$  which we are

\*Wait[34] shows that a more general expression for the normalized surface impedance at grazing incidence above a dielectric surface is  $\Delta = 1/\sqrt{\epsilon} (1 - 1/\epsilon)^{1/2}$ ; this expression is exact, and (9) is an approximation valid where  $|1/\epsilon_r|$  is much less than unity, as is true here.

†This conductivity depends upon the particular ocean and its temperature. It almost always lies between 3 and 5 mhos/meter. For this report, a value 4 mhos/meter will be used.



seeking for a statistically rough sea surface. This average, or effective,  $\bar{\Delta}$ , can then be used in the Norton-Sommerfeld formulation for ground-wave propagation, in the spirit of Wait's discussion<sup>[33]</sup>.

#### D. Guided Wave at Slightly Rough Impedance Boundary

##### 1. Description of Perturbed Surface Height

Let the mean plane of the sea surface be taken as the x-y plane. Then the z-coordinate (or height) to any surface point will be designated  $\zeta$ . See Figure 2. In general,  $\zeta$  is a function of x and y. It can now be expanded in a two-dimensional Fourier series over a square area of side length L.<sup>\*</sup> Thus

$$\zeta(x,y) = \sum_{m,n=-\infty}^{\infty} P(m,n) \exp\{ia(mx + ny)\} \quad , \quad (11)$$

where  $a = \frac{2\pi}{L}$ .  $P(m,n)$  is the Fourier coefficient of the m,n<sup>th</sup> spatial harmonic of the surface. The real nature of the surface height,  $\zeta(x,y)$ , requires that  $P^*(m,n) = P(-m,-n)$ , where  $P^*$  denotes the complex conjugate of P. This expansion can be employed both for deterministic surface-height profiles, and for random surface heights.

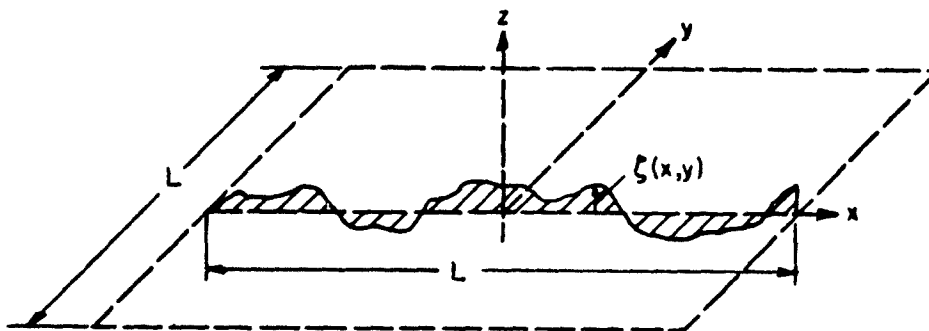


Figure 2. Slightly Rough Surface Geometry

<sup>\*</sup>The notation of Rice<sup>[23]</sup> will be retained here as much as possible in order to facilitate reference to this classical treatment.

In the case of the ocean,  $\zeta(x,y)$  is a random variable. Over a period of time, the surface profile undergoes a complete change, and we can think of an ensemble of random height profiles valid at different points in time. Hence, an average over time can be physically justified as representing an ensemble average, without undue discussion of the ergodic theorem. Therefore, averages (denoted here by  $\langle f \rangle$  or  $\bar{f}$ ) will be understood to be ensemble averages.

Since we shall have occasion to perform averages later, some definitions will be stated. These also facilitate the transition from a Fourier series with discrete surface-height spectral components to a continuous, average spectrum of surface heights. The reader is referred to Rice<sup>[23]</sup> for more detail on these definitions. It can be shown in any statistical treatment of random noise (e.g., see Davenport and Root<sup>[43]</sup> or Rice's earlier work<sup>[44]</sup>) that a Fourier series such as (11) can be used to validly represent a random variable. Furthermore, they show that the series coefficients,  $P(m,n)$ , become uncorrelated as  $L \rightarrow \infty$ . Practically, it is necessary that  $L$  be considerably larger than the surface correlation length or radius in order for the coefficients to appear uncorrelated; this condition is assumed satisfied here, since the ocean area considered here includes many ocean waves. Recalling now that the coordinate system was chosen so that  $\langle \zeta(x,y) \rangle = 0$ , we can state that

$$\langle P(m,n) \rangle = 0 \quad (12a)$$

$$\langle P(m,n) P(u,v) \rangle = \begin{cases} 0 & \text{for } u,v \neq -m,-n \\ \frac{\pi^2}{L^2} W(p,q) & \text{for } u,v = -m,-n \end{cases} \quad (12b)$$

where  $p = am = 2\pi m/L$  and  $q = an = 2\pi n/L$ . The function  $W(p,q)$  defines the average roughness spectral density of the surface, and  $p,q$ , are the radian wavenumbers (or spatial frequencies) along the  $x$ - and  $y$ -directions respectively. Using these equations, the following relationships are established.

$$\begin{aligned} \langle \zeta^2(x,y) \rangle &= \sum_{m,n,u,v} \langle P(m,n) P(u,v) \rangle e^{iax(m+u) + iay(n+v)} \\ &= \sum_{m,n} \langle P(m,n) P(-m,-n) \rangle \rightarrow \int_{-\infty}^{\infty} dm \int_{-\infty}^{\infty} dn \frac{\pi^2}{L^2} W(p,q) \end{aligned}$$

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$$= \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(p,q) dp dq = \sigma^2, \quad (13a)$$

$$\begin{aligned} \langle \zeta(x,y) \zeta(x',y') \rangle &= \sum_{m,n,u,v} \langle P(m,n) P(u,v) \rangle e^{iamx + iaux' + iany + iavy'} \\ &= \sum_{m,n} \langle P(m,n) P(-m,-n) \rangle e^{iam(x-x') + ian(y-y')} \\ &= \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(p,q) e^{ip\tau_x + iq\tau_y} dp dq = \sigma^2 R(\tau_x, \tau_y) \end{aligned} \quad (13b)$$

where  $\tau_x = x - x'$  and  $\tau_y = y - y'$ . The quantity  $\sigma^2$  is the mean-square height of the surface, and  $R(\tau_x, \tau_y)$  is the surface height correlation coefficient. Relationship (13b) merely states that the roughness height spectral density and surface height correlation function are Fourier transforms.

## 2. Description of Perturbed Fields of the Guided Wave

The underlying philosophy behind the perturbation approach to be used here requires that we expand the perturbed fields in the same eigenfunctions as those used for the perturbed surface. Such field solutions will be required to satisfy the wave equation. This leads us to choose the following form for our perturbed fields:

$$E_x = \Delta E(h,0,z) + \sum_{m,n=-\infty}^{\infty} A_{mn} E(m+h,n,z), \quad (14a)$$

$$E_y = \sum_{m,n=-\infty}^{\infty} B_{mn} E(m+h,n,z), \quad (14b)$$

$$E_z = E(h,0,z) + \sum_{m,n=-\infty}^{\infty} C_{mn} E(m+h,n,z), \quad (14c)$$

where

$$E(m+h,n,z) = E_0 \exp\{ia(h+m)x + iany + ib(m+h,n)z\}, \quad (15)$$

and

$$b(h + m, n) = \sqrt{k_0^2 - a^2(m + h)^2 - a^2 n^2} \quad (13)$$

The definition of  $b$  above is such that Equations (14a,b,c) satisfy the wave equation. The Cartesian components of the H-field are not given here, but are readily determined from Maxwell's equations.

In the above equations, the presence of roughness manifests itself as the summation terms. As the roughness height approaches zero,  $A_{mn}$ ,  $B_{mn}$ ,  $C_{mn}$ , will vanish, and  $b(h, 0) \rightarrow -k_0 \Delta$ ; Equations (14a,b,c) then become identical with Equations (7a,b,c) for a guided wave over a smooth impedance boundary.

Also,  $C_{00}$  is taken to be identically zero in (14c). This choice is possible; what it really means is that all of the remaining constants are normalized so that the 0,0 mode appearing in (14c) is removed from the summation as the first term,  $E(h, 0, z)$  with amplitude  $E_0$ . Physically, the guided-wave portions of the field appearing in Equations (14a,b,c) are all terms having the  $E(h, 0, z)$  structure. These are

$$E_x^G = (\Delta + A_{00}) E(h, 0, z) \quad , \quad (17a)$$

$$E_y^G = B_{00} E(h, 0, z) \quad , \quad (17b)$$

$$E_z^G = E(h, 0, z) \quad . \quad (17c)$$

The remaining portions of Equations (14a,b,c) consist of modes generated by the roughness. These modes, to be termed the scattered field here, actually include both propagating and evanescent modes.

### 3. The Average Normalized Impedance

The guided-wave portions of the perturbed fields are given by Equations (17a,b,c). When one compares (17a) and (17c) with Equations (7b) and (7a) for a smooth impedance boundary, one is led to define an "effective" or average impedance guided-wave propagation across a roughened surface as

$$\bar{\Delta} = \langle \Delta + A_{00} \rangle = \Delta + \langle A_{00} \rangle \quad , \quad (18)$$

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i.e., the effective impedance consists of the constant impedance of a smooth interface plus  $\langle A_{00} \rangle$ , which accounts for the roughness. This convenient definition will in fact be used, and the goal of the analysis will be to derive an expression for  $\langle A_{00} \rangle$ . If the roughness is to contribute to the impedance, we expect this average to be nonzero. Physically, (18) says that the average wave front and polarization tilt at the surface is  $\langle E_x^G \rangle / \langle E_z^G \rangle = \bar{\Delta} = \Delta + \langle A_{00} \rangle$ , which is another way of defining the effective surface impedance.

Since there is no  $E_y$  component of a guided wave over a smooth surface, we should expect that  $\langle B_{00} \rangle$  appearing in (17b) will be zero, meaning that  $\langle E_y^G \rangle$  is zero, on the average. This will in fact be shown later in the analysis.

#### 4. The Perturbed Leontovich Boundary Condition

Basically, the method of solution to the problem may be summarized as follows: We intend to employ Equations (14a,b,c), (which are solutions to the wave equation) along with the description of the surface given in (11), to determine the unknown constants  $A_{mn}$ ,  $B_{mn}$ ,  $C_{mn}$ , and  $h$ , by forcing the solution to satisfy the boundary condition at the perturbed surface. Of these constants, only  $A_{00}$  is really needed to determine the effective surface impedance, as shown in (18). The others will prove to be useful, however, in understanding the mechanism of scatter from the guided wave by the roughness.

The Leontovich boundary condition, as expressed in Equation (3), is written in terms of the  $\vec{E}$  and  $\vec{H}$ -fields at the surface, along with the unit normal to the surface,  $\hat{n}$ . All of these quantities vary with position along the surface. To reduce this equation to a usable form, however, we must express the unit normal in terms of the surface slopes; these are the partial derivatives of the surface height as a function of the independent variables,  $x$  and  $y$ . Defining them as

$$\zeta_x \equiv \frac{\partial \zeta(x,y)}{\partial x}, \quad \zeta_y \equiv \frac{\partial \zeta(x,y)}{\partial y}, \quad (19)$$

this normal is expressed in Cartesian coordinates as follows:

$$\hat{n} = \frac{-\zeta_x \hat{x} - \zeta_y \hat{y} + \hat{z}}{\sqrt{1 + \zeta_x^2 + \zeta_y^2}}, \quad (20)$$

where  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  are unit vectors along the coordinate axes. Although implicit,  $\zeta_x$  and  $\zeta_y$  are local functions of position,  $x$  and  $y$ , just as is  $\zeta(x,y)$ .

With the quadratic function in the denominator, Equation (20) is not very useful in a perturbation analysis. In order to be useful, a further restriction on the surface slopes must be made; they must be small, so that  $\zeta_x^2, \zeta_y^2 \ll 1$ . When such is the case, the quadratic can be simplified with the binomial expansion as follows  $[1 + \zeta_x^2 + \zeta_y^2]^{-1/2} \approx 1 - \frac{1}{2}(\zeta_x^2 + \zeta_y^2)$ , retaining only terms to the second order.

We shall employ the above approximation, and substitute (20) for  $\hat{n}$  into (3). In addition, we shall express  $\bar{H}$  in terms of  $\bar{E}$  by using Maxwell's equation. Then Equation (3) in vector form can be reduced to three scalar equations for the  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  components. We preserve terms in  $\zeta_x, \zeta_y$  up to the second order, i.e., up to  $\zeta_x^2$  and  $\zeta_y^2$ . The results for the  $\hat{x}$  and  $\hat{y}$  components are, respectively,

$$E_x - \zeta_x^2 E_x - \zeta_x \zeta_y E_y + \zeta_x E_z = \frac{\Delta}{ik_0} \left\{ -\zeta_y \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) - \left[ 1 - \frac{1}{2}(\zeta_x^2 + \zeta_y^2) \right] \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \right\} \quad (21a)$$

$$E_y - \zeta_y^2 E_y - \zeta_x \zeta_y E_x + \zeta_y E_z = \frac{\Delta}{ik_0} \left\{ -\zeta_x \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) - \left[ 1 - \frac{1}{2}(\zeta_x^2 + \zeta_y^2) \right] \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) \right\} \quad (21b)$$

The equation for the  $\hat{z}$  component is not written above and is not used in the analysis. The reason is that it is not independent of the other two, but can be obtained from them using the divergence equation.

Also, no attempt is made in the above equations to order the fields. For instance  $E_y$  (the  $y$ -component at the surface) will be small compared to  $E_z$ , i.e., it will be at least first-order if  $E_z$  is zero-order. Hence, the term  $\zeta_x \zeta_y E_y$  will be third-order and could have been omitted from the above equation, since only terms up to and including second-order are to be ultimately retained. However, the field components are not ordered as to smallness at this point so that the possibility of omitting significant terms will not occur. They can be easily dropped later, and the intervening analysis will serve as proof that they are small.

This boundary condition, as noted previously, applies at the perturbed surface, relating the various  $E$ -field components and their derivatives.

### 5. Restrictions Made and the Perturbation Parameters

In many analyses, so many assumptions and approximations are made along the way that the reader is never sure of the validity of the results. In summary, we shall state at this point the assumptions we have made.

(i) The normalized surface impedance,  $\Delta$ , is considerably less than unity. This is true for sea water, and at the highest frequency we intend to consider (i.e., 100 MHz),  $|\Delta| \approx .038$ ; it decreases further at lower frequencies.

(ii) The radii of curvature of the ocean waves producing scatter is considerably greater than the wavelength within the medium. At low frequencies, this restriction is the most serious, and at 10 MHz, for example, it was shown in a previous section that it is very adequately fulfilled.

(iii) In free space,  $(k_0\zeta)^2 \ll 1$ , i.e., the surface height compared to wavelength is small. In very high seas, say Sea State 6, where wave heights,  $\zeta$ , can be  $\pm 4$  feet from a mean level,  $(k_0\zeta)^2 \approx .4$  at 30 MHz. For calmer seas, which are much more prevalent, the frequency of validity can be extended higher; e.g., to 100 MHz for Sea State 3.

(iv)  $\zeta_x^2, \zeta_y^2 \ll 1$ , i.e., the ocean waves responsible for scatter have relatively small slopes. This is adequately satisfied for deep seas (i.e., away from coastal reef areas which can produce breakers) where the surface angle for waves whose lengths are several meters will rarely exceed  $20^\circ$ , for which  $\zeta_x^2 < .14$ .

It is important to note that the above conditions can even fail at a finite number of points on the surface, and the validity of the results is not seriously impaired. They cannot fail, on the average, over most of the surface, however. Because of the nature of the perturbation analysis which we intend to employ, we will have need to "order" or expand terms mathematically about "smallness" or "perturbation" parameters. As a result of the above restrictions, we shall employ the following perturbation parameters:

- (i)  $k_0\zeta$ ; we shall include terms to second order in this parameter
- (ii)  $\zeta_x$  and  $\zeta_y$ ; likewise, terms to second order will be retained
- (iii)  $\Delta$ ; we shall retain terms to first order in this parameter
- (iv)  $A_{mn}, B_{mn}, C_{mn}$ ; these are taken to be of at least first-order in

smallness, since they represent field perturbation due to the roughness. On the basis of the restrictions mentioned previously, these parameters will be shown to be at least first-order. However, in order to facilitate the derivation, we shall employ the definition  $A_{mn} = A_{mn}^{(1)} + A_{mn}^{(2)} + \dots$ , with similar definitions for  $B_{mn}$ ,  $C_{mn}$ . This means that we break  $A_{mn}$  up into first-order contributions, second-order contributions, and so on. The first-order contributions to  $A_{mn}$  (i.e.,  $A_{mn}^{(1)}$ ) then come from first-order terms in  $k_0\zeta$ ,  $\zeta_x$ ,  $\zeta_y$ , and  $\Delta$ .

6. Derivation of First-Order Coefficients  
of the Perturbed Field:  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$

Equations (14a,b,c) represent the perturbed field expressed in terms of the coefficients  $A_{mn}$ ,  $B_{mn}$ , and  $C_{mn}$ . These coefficients we order according to smallness, as discussed in the preceding paragraph. Then the resulting expressions for the perturbed fields are substituted into the boundary conditions (21) and evaluated at the boundary, i.e., for  $z = \zeta$ . At this point, we also discard terms obviously of higher order than second. We employ the following expansions of the fields at the surface, correct to the second order in  $\zeta$ :

$$E(m+h, n, z) \Big|_{z=\zeta} \\ = E(m+h, n, \zeta) \approx F(h) E_x(m, n) \left[ 1 + ib(m+h, n)\zeta - \frac{1}{2} b^2(m+h, n)\zeta^2 \right], \quad (22a)$$

$$\frac{\partial E(m+h, n, z)}{\partial x} \Big|_{z=\zeta} \approx \\ \approx i(h+m)a F(h) E_x(m, n) \left[ 1 + ib(m+h, n)\zeta - \frac{1}{2} b^2(m+h, n)\zeta^2 \right], \quad (22b)$$

$$\frac{\partial E(m+h, n, z)}{\partial y} \Big|_{z=\zeta} \approx \\ \approx ina F(h) E_x(m, n) \left[ 1 + ib(m+h, n)\zeta - \frac{1}{2} b^2(m+h, n)\zeta^2 \right], \quad (22c)$$



$$\left. \frac{\partial E(m+h, n, z)}{\partial z} \right|_{z=\zeta} =$$

$$= ib(m+h, n) F(h) \text{Ex}(m, n) \left[ 1 + ib(m+h, n)\zeta - \frac{1}{2} b^2(m+h, n)\zeta^2 \right], \quad (22d)$$

where  $F(h) = E_0 e^{ihax}$ , and  $\text{Ex}(m, n) = \exp[iamx + iany]$ .

When the above expansions are used, the following expressions are obtained from (21), correct to the second order in the smallness parameters defined in the preceding section.

$$\begin{aligned} & \Delta F(h) \left[ 1 + ib(h, 0)\zeta - \frac{1}{2} b^2(h, 0)\zeta^2 \right] + \\ & + \sum \left\{ \left[ A_{mn}^{(1)} + A_{mn}^{(2)} \right] \left[ 1 - ib(m+h, n)\zeta - \frac{1}{2} b^2(m+h, n)\zeta^2 \right] \right\} \cdot \\ & \cdot F(h) \text{Ex}(m, n) - \zeta_x^2 \Delta F(h) + \zeta_x F(h) \left[ 1 + ib(h, 0)\zeta \right] + \zeta_x \sum C_{mn}^{(1)} F(h) \text{Ex}(m, n) = \\ & = \frac{\Delta}{ik_0} \left[ -\zeta_y \sum \left\{ B_{mn}^{(1)} i(m+h)a - A_{mn}^{(1)} ina \right\} F(h) \text{Ex}(m, n) - \right. \\ & \quad \left. - ib(h, 0) \Delta F(h) \left[ 1 + ib(h, 0)\zeta - \frac{1}{2} b^2(h, 0)\zeta^2 \right] + \right. \\ & \quad \left. + iha F(h) \left[ 1 + ib(h, 0)\zeta - \frac{1}{2} b^2(h, 0)\zeta^2 \right] - \sum \left\{ \left[ \left( A_{mn}^{(1)} + A_{mn}^{(2)} \right) ib(m+h, n) - \right. \right. \right. \\ & \quad \left. \left. \left. - \left( C_{mn}^{(1)} + C_{mn}^{(2)} \right) i(m+h)a \right] \right\} \right. \\ & \quad \left. \cdot \left[ 1 + ib(m+h, n)\zeta \right] \right\} F(h) \text{Ex}(m, n) + \frac{1}{2} \left( \zeta_x^2 + \zeta_y^2 \right) \left[ ib(h, 0)\Delta - iha \right] F(h) \quad (23a) \\ & \quad \sum \left\{ \left[ B_{mn}^{(1)} + B_{mn}^{(2)} \right] \left[ 1 + ib(m+h, n)\zeta \right] \right\} F(h) \text{Ex}(m, n) - \zeta_x \zeta_y \Delta F(h) + \\ & \quad + \zeta_y F(h) \left[ 1 + ib(h, 0)\zeta \right] \\ & + \zeta_y \sum C_{mn}^{(1)} F(h) \text{Ex}(m, n) = \frac{\Delta}{ik_0} \left[ -\zeta_x \sum \left\{ A_{mn}^{(1)} ina - B_{mn}^{(1)} i(m+h)a \right\} F(h) \text{Ex}(m, n) \right. \\ & \quad \left. - \sum \left\{ \left[ B_{mn}^{(1)} + B_{mn}^{(2)} \right] ib(m+h, n) - \left( C_{mn}^{(1)} + C_{mn}^{(2)} \right) ina \right] \left[ 1 + ib(m+h, n)\zeta \right] \right\} \\ & \quad F(h) \text{Ex}(m, n) \quad (23b) \end{aligned}$$

In the above equations, the factor  $F(h)$  is common to all terms and can be dropped.

Following the classical perturbation technique, we now collect terms of each order and set each set separately equal to zero. Let us collect first the zero-order terms from (23a) (there are no zero-order terms in (23b)).

$$\Delta + \frac{\Delta}{ik_0} [ib(h,0)\Delta - iha] = 0 \quad (24)$$

This equation contains the propagation constants associated with the guided wave. In order to convince ourselves that the left side is really equal to zero (to at least the second order), let us employ the divergence condition on the guided-wave fields as expressed in (17).

$$\nabla \cdot \overline{E^G} = 0 = \frac{\partial E_x^G}{\partial x} + \frac{\partial E_y^G}{\partial y} + \frac{\partial E_z^G}{\partial z} = iha(\Delta + A_{00}) + ib(h,0) = 0 \quad ,$$

$$\therefore b(h,0) = -ha(\Delta + A_{00}) \quad (25)$$

But,  $ha$  is defined in terms of  $b(h,0)$  from (16), i.e.,  $ha = \sqrt{k_0^2 - b^2(h,0)}$ . The quantity  $b(h,0)$  will be small compared to  $k_0$  since it represents the  $z$ -portion of the wavenumber for the guided wave. On the other hand,  $ha$  will be very close to  $k_0^*$ . Hence

$$ha = k_0 \left( 1 - \frac{1}{2} \frac{b^2(h,0)}{k_0^2} + \dots \right) \quad (26)$$

by the binomial expansion. Now, let us substitute (25) for  $b(h,0)$  into (24).

$$\Delta - \frac{ha}{k_0} \Delta [\Delta^2 + A_{00}\Delta + 1] = 0 \quad .$$

Now, substitute (26) for  $ha$  into the above. The result is

$$\Delta^3 + \Delta^2 A_{00} - \frac{1}{2} \frac{b^2(h,0)}{k_0^2} \Delta = 0 \quad (27)$$

All of the terms in the above equation are of higher order than second in  $\Delta$ ,  $A_{00}$ , and  $b(h,0)$ . Hence, Equation (24) for the zero-order terms is actually zero to the order of terms to be retained throughout this analysis.

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\*These statements are true for the types of surfaces being considered here, i.e., those which satisfy the restrictions imposed in the preceding section.

Now, let us collect all of the terms from (23a) and (23b) which are of first-order in  $k_0 \zeta$ ,  $\zeta_x$ ,  $\zeta_y$ ,  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$ . They are:

$$i \Delta b(h,0) \zeta + \zeta_x + \frac{\Delta^2}{k_0} i b^2(h,0) \zeta - \frac{\Delta}{k_0} i h a b(h,0) \zeta + \sum \left\{ \left[ 1 + \frac{\Delta}{k_0} b(m+h,n) \right] A_{mn}^{(1)} - \frac{\Delta}{k_0} (m+h)a C_{mn}^{(1)} \right\} Ex(m,n) = 0 \quad (28a)$$

$$\zeta_y + \sum \left\{ \left[ 1 + \frac{\Delta}{k_0} b(m+h,n) \right] B_{mn}^{(1)} - \frac{\Delta}{k_0} n a C_{mn}^{(1)} \right\} Ex(m,n) = 0 \quad (28b)$$

In equation (28a) above, the terms multiplying  $i b(h,0) \zeta$  are identically the same terms as appeared in Equation (24); they are equal to zero, at least out beyond the second order. Hence, Equation (28a) becomes

$$\zeta_x + \sum \left\{ \left[ 1 + \frac{\Delta}{k_0} b(m+h,n) \right] A_{mn}^{(1)} - \frac{\Delta}{k_0} (m+h)a C_{mn}^{(1)} \right\} Ex(m,n) = 0 \quad (28c)$$

The surface slopes  $\zeta_x$  and  $\zeta_y$  can be written as a series in the eigenfunctions  $Ex(m,n)$  by differentiating Equation (11):

$$\zeta_x = \sum i m a P(m,n) Ex(m,n); \quad \zeta_y = \sum i n a P(m,n) Ex(m,n) \quad (29)$$

When these expressions are substituted into (28b) and (28c), one has a series whose terms each contain the same eigenfunction  $Ex(m,n)$  as a factor. In order for such a series to be zero, each term must be identically equal to zero\*. The following two equations are obtained from this procedure.

$$\left[ 1 + \frac{\Delta b(m+h,n)}{k_0} \right] A_{mn}^{(1)} - \frac{\Delta (m+h)a}{k_0} C_{mn}^{(1)} = -i m a P(m,n) \quad (30a)$$

$$\left[ 1 + \frac{\Delta b(m+h,n)}{k_0} \right] B_{mn}^{(1)} - \frac{\Delta n a}{k_0} C_{mn}^{(1)} = -i n a P(m,n) \quad (30b)$$

Represented above are two equations in three unknowns,  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$ . To this we can add a third independent equation by using the divergence condition (i.e.,  $\nabla \cdot \vec{E} = 0$ ) with Equations (14a,b,c). The  $m,n = 0$  terms give (25). All higher terms when equated to zero give the following identity:

$$(m+h)a A_{mn}^{(1)} + n a B_{mn}^{(1)} + b(m+h,n) C_{mn}^{(1)} = 0 \quad (30c)$$

\*This result can be proved formally by integrating the series over the range of  $x$  and  $y$ , i.e., from  $-L/2 \leq x, y \leq L/2$ .

Now we have three equations in three unknowns. They are most easily solved by expressing  $C_{mn}^{(1)}$  in terms of  $A_{mn}^{(1)}$  and  $B_{mn}^{(1)}$  using (30c). This then when substituted into (30a) and (30b) leaves two equations in two unknowns, which are then readily solved by elimination. The final results for the first-order perturbed field coefficients are:

$$A_{mn}^{(1)} = \frac{N_A}{D} P(m,n); \quad B_{mn}^{(1)} = \frac{N_B}{D} P(m,n); \quad C_{mn}^{(1)} = \frac{-(m+h)aN_A - naN_B}{b(m+h,n)D} P(m,n), \quad (31)$$

where

$$N_A = -ina \left[ 1 + \Delta \left( \frac{b(m+h,n)}{k_0} + \frac{n^2 a^2}{k_0 b(m+h,n)} \right) \right] + ina \frac{\Delta(m+h)na^2}{k_0 b(m+h,n)}, \quad (32a)$$

$$N_B = -ina \left[ 1 + \Delta \left( \frac{b(m+h,n)}{k_0} + \frac{(m+h)^2 a^2}{k_0 b(m+h,n)} \right) \right] + ima \frac{\Delta(m+h)na^2}{k_0 b(m+h,n)}, \quad (32b)$$

$$D = \left[ 1 + \Delta \left( \frac{b(m+h,n)}{k_0} + \frac{(m+h)^2 a^2}{k_0 b(m+h,n)} \right) \right] \left[ 1 + \Delta \left( \frac{b(m+h,n)}{k_0} + \frac{n^2 a^2}{k_0 b(m+h,n)} \right) \right] - \frac{\Delta^2 (m+h)^2 n^2 a^4}{k_0^2 b^2 (m+h,n)}. \quad (32c)$$

Several facts are worth noting in the above equations for the first-order coefficients of the perturbed fields. First of all, the quantities  $N_A$ ,  $N_B$ , and  $D$  contained in the factor before  $P(m,n)$  are functions of the scattered-field propagation directions and the surface material properties; the former directions are determined by  $(m+h)a$  and  $na$ , while the surface material properties are represented entirely by the normalized surface impedance,  $\Delta = \sqrt{\frac{\mu}{\epsilon}} \frac{Z}{\eta_0}$ . These quantities do not depend upon the surface roughness profile. The dependence upon this profile is contained entirely in the factor  $P(m,n)$ . The latter, recall, is the  $m,n$ -th Fourier coefficient in the expansion for the surface height, as expressed in (11). Hence, the strength of the scattered plane wave in the direction defined by  $(m+h)a$ ,  $na$ , is directly proportional to the  $m,n$ -th Fourier component of the surface height (to the first order). If a surface-height component corresponding to  $m,n$  is zero, then  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$  are zero, meaning that no scattered wave propagates in the direction defined by  $(m+h)a$ ,  $na$ . The theory of scatter from slightly-rough periodic surface dates back to Lord Rayleigh<sup>[45]</sup>. He derived expressions similar to (31) and noted that scatter took place in unique directions, or lobes, determined by the Fourier components of the surface and the length of their periods with respect to the wavelength.

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Let us recall that the object of the present analysis is the derivation of  $A_{00}$ ; it is the average of this coefficient which contributes to the surface impedance, as seen in (18). To the first order, however,  $A_{00}^{(1)}$  is zero. This is true because  $A_{00}^{(1)}$  is directly proportional to  $P(0,0)$ , i.e., the DC or mean height of the surface  $\zeta(x,y)$  above the x-y plane. The x-y plane was chosen to be the mean plane, however, and hence  $P(0,0)$  is identically zero. Thus we see that  $A_{00}$  can be no greater than second order. It is for this reason that second-order terms were retained in the equations.

#### 7. Derivation of the Second-Order Coefficient $A_{00}^{(2)}$

As shown in the preceding section, the contribution of the roughness to the surface impedance is contained in  $A_{00}^{(2)}$ ;  $A_{00}^{(1)}$  is identically zero. Hence, we shall determine  $A_{00}^{(2)}$  in this section from Equations (23). While it is possible to determine all of the general second-order coefficients,  $A_{mn}^{(2)}$ ,  $B_{mn}^{(2)}$ ,  $C_{mn}^{(2)}$ , this will not be done here; they are not necessary for our purposes.

It turns out that  $A_{00}^{(2)}$  can be determined entirely from (23a). Keeping with the perturbation technique, let us gather the second-order terms in (23a) and equate them to zero.

$$\begin{aligned}
 & -\frac{1}{2} b^2(h,0) \zeta^2 \left[ \Delta - \frac{\Delta^2}{k_0} b(h,0) + \frac{\Delta}{k_0} ha \right] + \sum \left[ ib(m+h,n) \zeta A_{mn}^{(1)} + A_{mn}^{(2)} \right] Ex(m,n) \\
 & - \Delta \zeta_x^2 + ib(h,0) \zeta \zeta_x + \zeta_x \sum C_{mn}^{(1)} Ex(m,n) - \frac{\Delta}{ik_0} \left[ -\zeta_y \sum \left\{ B_{mn}^{(1)} i(m+h)a \right. \right. \\
 & \left. \left. - A_{mn}^{(1)} ina \right\} Ex(m,n) - \sum \left\{ ib(m+h,n) \zeta \left[ A_{mn}^{(1)} ib(m+h,n) - C_{mn}^{(1)} i(m+h)n \right] + \left[ A_{mn}^{(2)} ib(m+h,n) \right. \right. \right. \\
 & \left. \left. \left. - C_{mn}^{(2)} i(m+h)n \right] \right\} Ex(m,n) + \frac{1}{2} (\zeta_x^2 + \zeta_y^2) (ib(h,0)\Delta - iha) \right] . \quad (33)
 \end{aligned}$$

The first term containing  $\zeta^2$  is zero because the factor in square brackets is zero, as seen from (24). Also, using (24), the last term can be simplified, since the factor  $ib(h,0)\Delta - iha$  is  $-ik_0$ .

Now, let us group the unknown, second-order coefficients  $A_{mn}^{(2)}$  and  $C_{mn}^{(2)}$  appearing in (33) together on the left side of the equation. The result is

$$\begin{aligned}
& \sum \left[ \left( 1 + \Delta \frac{b(m+h,n)}{k_0} \right) A_{mn}^{(2)} - \Delta \frac{(m+h)a}{k_0} C_{mn}^{(2)} \right] \text{Ex}(m,n) = \frac{\Delta}{2} (\zeta_x^2 - \zeta_y^2) + ik_0 \Delta \zeta \zeta_x \\
& - \sum \left\{ \left[ ib(m+h,n) \zeta \left( 1 + \Delta \frac{b(m+h,n)}{k_0} \right) - \Delta \frac{na}{k_0} \zeta_y \right] A_{mn}^{(1)} + \Delta \frac{(m+h)a}{k_0} \zeta_y B_{mn}^{(1)} \right. \\
& \left. + \left[ -ib(m+h,n) \zeta \Delta \frac{(m+h)a}{k_0} + \zeta_x \right] C_{mn}^{(1)} \right\} \text{Ex}(m,n) . \quad (34)
\end{aligned}$$

The right side of the above equation will involve double summation sets. In order to employ the orthogonality relationship of the eigenfunctions,  $\text{Ex}(m,n)$ , more effectively it is convenient to rearrange the double summation. The examples below illustrate the procedure.

$$\begin{aligned}
\zeta_x^2 &= \sum_{\alpha, \beta} \sum_{\gamma, \zeta} (i\alpha m) (i\gamma a) P(\alpha, \beta) P(\gamma, \zeta) \text{Ex}(\alpha + \gamma, \beta + \zeta) \\
&= \sum_{m, n} \sum_{h, l} ia(m-h) (ia h) P(h, l) P(m-h, n-l) \text{Ex}(m, n) , \quad (35a)
\end{aligned}$$

$$\begin{aligned}
\sum_{m, n} ib(m+h, n) C_{mn}^{(1)} &= \sum_{\alpha, \beta} \sum_{\gamma, \zeta} ib(\alpha+h, \beta) P(\gamma, \zeta) A_{\alpha\beta}^{(1)} \text{Ex}(\alpha + \gamma, \beta + \zeta) \\
&= \sum_{m, n} \sum_{h, l} ib(h+h, l) A_{hl}^{(1)} P(m-h, n-l) \text{Ex}(m, n) . \quad (35b)
\end{aligned}$$

Likewise, the remainder of the terms on the right side can be arranged so that the factor  $\text{Ex}(m,n)$  appears explicitly. As a result, the right side of (34) then becomes:

$$\begin{aligned}
& \frac{\Delta}{2} \sum \sum \left[ (ia h) ia(m-h) - (ia l) ia(n-l) \right] P(h, l) P(m-h, n-l) \text{Ex}(m, n) \\
& + ik_0 \Delta \sum \sum ia(m-h) P(h, l) P(m-h, n-l) \text{Ex}(m, n) \\
& - \sum \sum \left\{ A_{hl}^{(1)} \left[ ib(h+h, l) + \Delta \frac{ib^2(h+h, l)}{k_0} - \Delta \frac{ia^2 l(n-l)}{k_0} \right] + B_{hl}^{(1)} \Delta \frac{ia^2 (h+h)(n-l)}{k_0} \right. \\
& \left. + C_{hl}^{(1)} \left[ ia(m-h) - \Delta \frac{ib(h+h, l)a(h+h)}{k_0} \right] \right\} P(m-h, n-l) \text{Ex}(m, n) . \quad (36)
\end{aligned}$$

Recall, we are interested in finding  $A_{00}^{(2)}$ , i.e., the second-order coefficient for  $m = n = 0$ . This can be readily done by invoking the orthogonality of the  $Ex(m,n)$  functions over the square region of interest. As a result of this relationship, the coefficients of  $Ex(0,0)$  on each side of the equation are equal.

The left side of (34) becomes

$$\left(1 + \Delta \frac{b(h,0)}{k_0}\right) A_{00}^{(2)} - \Delta \frac{ha}{k_0} C_{00}^{(2)}.$$

This result can be simplified further. First of all,  $C_{00}$  is zero, by definition, to all orders; this definition is merely one possible normalization, as discussed after Equations (14). As seen above, it is a logical selection because it results in a convenient separation of coefficients. Secondly, the term  $\Delta \frac{b(h,0)}{k_0}$  is of second order compared to unity. This is evident from (25) where we see that  $\frac{b(h,0)}{ha} = \frac{b(h,0)}{k_0} = -\Delta - A_{00}^{(2)}$ . Hence, it can be neglected compared to unity, to the order of the analysis here. Then the left side becomes merely  $A_{00}^{(2)}$ .

When we group the terms on the right side of (34), as shown in (36), which multiply  $Ex(0,0)$ , we have the resulting equation for  $A_{00}^{(2)}$ .

$$A_{00}^{(2)} = \sum_{h,l} \left\{ \left[ -ib(h+h,l) A_{hl}^{(1)} + ia h C_{hl}^{(1)} \right] + \Delta \left[ k_0 a h + \frac{a^2 h^2 - a^2 l^2}{2} \right] P(h,l) \right. \\ \left. - \Delta \left[ \left( \frac{ia^2 l^2}{k_0} + \frac{ib^2(h+h,l)}{k_0} \right) A_{hl}^{(1)} + \frac{ia^2(h+h)l}{k_0} B_{hl}^{(1)} + \frac{ib(h+h,l)a(h+h)}{k_0} C_{hl}^{(1)} \right] \right\} P(-h,-l) \quad (37)$$

In the above equation, the terms in the first set of square brackets within the summation are the lowest-order terms in the surface impedance,  $\Delta$ . For a perfectly conducting rough surface,  $\Delta \rightarrow 0$ , and only these first terms are left. Hence, the remaining terms represent the contribution due to the finite conductivity of the surface material, correct to the first order in  $\Delta$ .

At this point, we can employ the expressions derived earlier for the first-order coefficients  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$ ; these are shown in Equations (31) and (32). When they are substituted into (37), considerable algebraic simplification is possible. Sparring the reader the details, we write the final expressions for the various terms.

In reducing the expression in the first set of square brackets, we preserve terms up to  $\Delta$ . The final result is

$$-ib(\hbar + h, l) A_{\hbar l}^{(1)} + ia\hbar C_{\hbar l}^{(1)} = \left[ \frac{k_0 a^2 \hbar^2}{b(\hbar + h, l) D(\hbar, l)} + \frac{\Delta}{D(\hbar, l)} (a^2 \hbar^2 + a^2 l^2) \right] P(\hbar, l), \quad (38a)$$

where  $D(\hbar, l)$  is given in (32c) with  $m, n$  there replaced by  $\hbar, l$ .

The expression in the second set of square brackets is already expressed in the form we desire. That in the third set of brackets can be simplified, and only terms of zero-order in  $\Delta$  need be retained because the expression is already of order  $\Delta$  due to this multiplicative factor. Thus to the lowest order, the expression within these third brackets becomes

$$\left[ \frac{k_0 a \hbar}{D(\hbar, l)} + O(\Delta) \right] P(\hbar, l). \quad (38b)$$

Using the simplifications expressed in (38), Equation (37) can thus be written as follows:

$$A_{00}^{(2)} = \sum_{\hbar, l} \left\{ \frac{k_0 a^2 \hbar^2}{b(\hbar + h, l) D(\hbar, l)} + \Delta \left[ \frac{a^2 \hbar^2 + a^2 l^2 - k_0 a \hbar}{D(\hbar, l)} + \left( k_0 a \hbar + \frac{a^2 \hbar^2 - a^2 l^2}{2} \right) \right] \right\} |P(\hbar, l)|^2. \quad (39)$$

In addition, the expression  $D(\hbar, l)$  can be reduced, and to order  $\Delta$ , it becomes

$$D(\hbar, l) \approx 1 + \frac{k_0}{b(\hbar + h, l)} \Delta \left[ \frac{b^2(\hbar + h, l)}{k_0^2} + 1 \right]. \quad (40)$$

Thus, we have obtained an expression for  $A_{00}^{(2)}$  in terms of a summation of terms in the square of the Fourier coefficients of the surface height. This constant, as shown in (18), is the contribution to the surface impedance due to the presence of the roughness. Since terms are retained to order  $\Delta$  in  $A_{00}^{(2)}$ , one can observe the effects of finite conductivity of the surface material on the roughness contribution. When the material is such that  $|\Delta|$  is large, Equation (39) is obviously inadequate to describe the effect of roughness. The second term in (39) provides a first-order correction in  $\Delta$  for moderately small values of this parameter.



### 8. Average Surface Impedance

Equation (18) expresses the overall effective surface impedance as the sum of  $\Delta$ , the constant impedance of the surface material in the absence of roughness, plus the average of  $A_{00}^{(2)}$ , which is the effective contribution due to roughness. Before proceeding to take the indicated average, it should be noted that there may be some cases where an average is not desired. Equation (39) expresses  $A_{00}^{(2)}$  exactly, in terms the coefficients of the Fourier expansion for the surface profile. If this surface height profile is known exactly, these coefficients can be determined exactly, and (39) represents a deterministic result\*. For example, if the surface profile is a pure, one-dimensional sinusoid, then  $\zeta(x)$  can be expanded so that only two terms in the series (11) are non-zero. In this case,  $A_{00}^{(2)}$  consists of only two terms, the one which has  $h = \pm 1$ ,  $l = 0$ .

The sea surface is an example in which the exact height profile is neither known deterministically nor constant over a very long period of time. Hence, it is one situation in which an average is the only meaningful description of the effects of the surface on propagation. To take the average of (39), we shall employ the definition of the height spectrum  $W(p,q)$  in terms of  $\langle |P(m,n)|^2 \rangle$ , as expressed in (12b). In addition, we allow the summation over  $h,l$  to approach an integral, in the same manner as was done in (13). Understanding here that  $p = \pi h = 2\pi h/L$ , and  $q = \pi l = 2\pi l/L$ , we obtain the following result for  $\langle A_{00}^{(2)} \rangle$ , and hence  $\bar{\Delta}$ :

$$\bar{\Delta} = \Delta + A_{00}^{(2)},$$

or

$$\bar{\Delta} = \Delta + \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(p,q) W(p,q) dp dq, \quad (41)$$

where

$$F(p,q) = \frac{p^2 + b' \Delta (p^2 + q^2 - k_0 p)}{b' + \Delta (b'^2 + 1)} + \Delta \left( \frac{p^2 - q^2}{2} + k_0 p \right), \quad (42a)$$

$$b' = \frac{1}{k_0} \sqrt{k_0^2 - (p + k_0)^2 - q^2}, \quad (42b)$$

\*The validity of the result is still dependent upon fulfillment of the restrictions described in Section 5 above, of course.

and  $W(p,q)$  is the two-dimensional surface height roughness spectrum; it is a function of the spatial wavenumbers  $p,q$  corresponding to the  $x,y$  directions and is defined in terms of the height correlation function in (13b). In obtaining the above expressions, we also used the approximation that  $h \sim k_0$ ; this is valid so long as  $\bar{\Delta}$  is small compared to unity, as seen from (26).

As a check on (41), if we permit  $\Delta$  to approach zero in (42a), we obtain  $F(p,q) = p^2/b'$ , and

$$\bar{\Delta} = \Delta + \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{p^2 W(p,q) dp dq}{b'} ; \quad (43)$$

this result was derived from an integral equation technique by Feinberg in 1944<sup>[20]</sup>, and repeated by Bremmer<sup>[8]</sup>. In addition, if we permit the surface to become perfectly conducting so that  $\Delta$  approaches zero everywhere, (43) consists only of the integral; this expression checks with that of Rice<sup>[23]</sup>, who initially assumed a perfectly conducting surface. Hence, we have in (41) an expression valid to order  $\Delta$  for imperfectly conducting, rough surfaces.

#### 9. Physical Interpretation of Roughness Contribution to Surface Impedance

Equations (41) and (43) show that the contribution of the roughness to the surface impedance is represented by the integral. To better understand the interaction of the guided wave with the roughness, let us consider first the simpler form of the integral, i.e., that in (43), which is valid when the surface is perfectly conducting. The height spectrum,  $W(p,q)$ , is always a positive real quantity. Thus the nature of the integral contribution depends entirely upon the denominator, i.e.,  $b' = \sqrt{1 - \left(\frac{p}{k_0} + 1\right)^2 - \left(\frac{q}{k_0}\right)^2}$ . This quantity can be either pure real and positive or purely imaginary and positive, depending upon whether  $\left(\frac{p}{k_0}\right)^2$  is less than or greater than unity.

This can be better illustrated by referring to Figure 3. Let us separate the contribution to  $\bar{\Delta}$  of the integral into two parts,  $R_{\Delta} - iX_{\Delta}$ . The contribution  $R_{\Delta}$  comes entirely from that part of the height spectrum lying within the unit circle centered at  $\frac{p}{k_0} = -1, \frac{q}{k_0} = 0$ . If there are no roughness spatial frequencies, or waves, within this region, then the resistive contribution is zero; these spatial frequencies

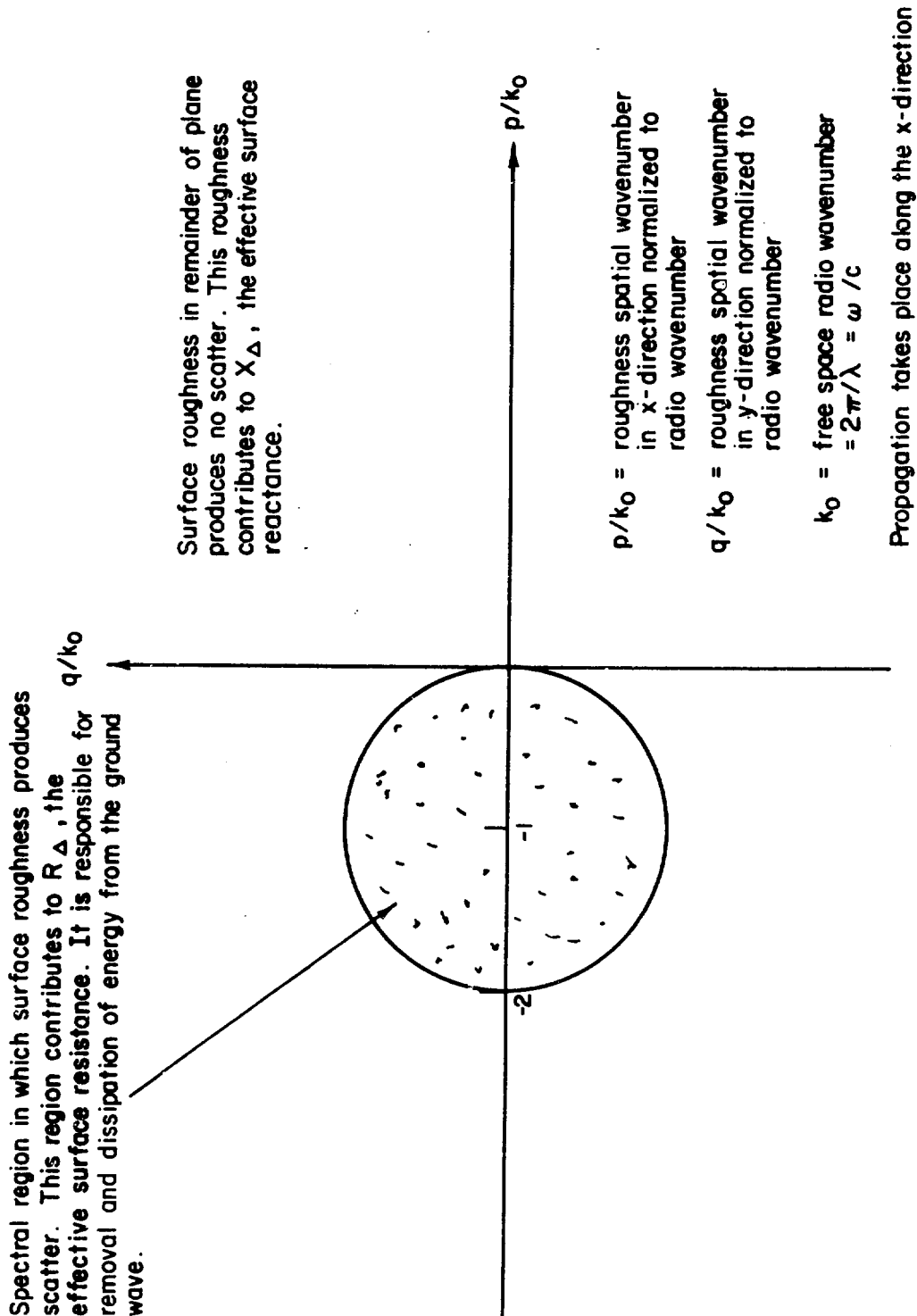


Figure 3. Effect of Various Regions of Spatial Roughness Spectrum on Effective Surface Impedance

must thus be less than  $2k_0$ . Physically, this statement means that only ocean waves longer than  $\frac{\lambda}{2}$  can contribute to the resistive portion. From a study of scatter from this type of surface<sup>[41,42]</sup>, it has been shown that waves whose lengths are greater than  $\lambda/2$  are responsible for scatter. Hence the interpretation of the resistive portion becomes clearer; longer ocean waves whose wavenumbers lie within the circle are responsible for removal of energy from the guided wave and scatter of this energy into all directions in the upper hemisphere. This energy removal produces an increase in the resistive term of the surface impedance.

On the other hand, if there are no ocean waves whose lengths are greater than a half-wavelength (i.e., which lie outside the unit circle), then the roughness contribution to  $\bar{A}$  is purely reactive; in addition, it is always an inductive reactance. From scatter theory<sup>[41,42]</sup>, roughness waves of these shorter lengths do not scatter (at least to the first order in  $k_0\zeta$ , the roughness height). Hence, this higher frequency roughness produces a perturbation on the local field at the surface which exists only at and near the region between the waves; since there are no scattered propagating fields removing energy from the guided wave, this effect should be evident only very near the surface. The perturbed modes in this case are not propagating, but evanescent, because the only coefficients  $A_{mn}^{(1)}$ ,  $B_{mn}^{(1)}$ , and  $C_{mn}^{(1)}$  which are non-zero are those whose wavenumbers  $a_m$ ,  $a_n$  are such that  $b(m+h, n)$  is imaginary, and hence, the exponential  $e^{ib(m+h, n)z}$  in the perturbed fields (Equations 14-16) is attenuating in the  $+z$  direction above the surface.

This reactive nature of surfaces with short roughness periods is confirmed by many studies on corrugated surfaces, and especially, surfaces with rectangular slots or ribs<sup>[46,33]</sup>. Elliott's analysis<sup>[46]</sup> is based upon Floquet's theorem (as are many other similar treatments), and employs an approximation of the field between the ribs. The approximation is valid, Elliott points out, when there are more than ten corrugation periods per radio wavelength. Wait<sup>[33]</sup> feels this requirement can be relaxed to five periods per wavelength. These analyses then show that the dominant guided mode has a wavenumber which can be related to an effective, purely reactive (inductive), surface impedance. The slots or ribs act as resonators in this case. The analysis we used here cannot be used to analyze such structures, because of the requirements we imposed in Section 5 above demands that the surface slopes be small. A ribbed surface has infinite surface slopes at the edges or walls of the ribs. Nonetheless, the two different techniques both lead to reactive surface impedances

when there are more than two surface periods per each radio wavelength. We shall see later that the reactances predicted by our analysis are well below those applicable to ribbed surfaces with similar heights but whose walls are vertical; such a result is expected, because more sloping sides will reduce the resonator effect between teeth.

When the surface is not perfectly conducting, finite fields can exist beneath the surface as well as above it. This fact is accounted for by the Leontovich boundary condition and the use of a non-zero surface impedance,  $\Delta \left( \Delta = \sqrt{\frac{\mu}{\epsilon}} \right)$ . One effect of this finite impedance is in evidence in the denominator of the integrals in (41); the denominator does not go to zero on the unit circle in Figure 3. This fact makes for easier numerical integration in the vicinity of this singularity. Results later will show, however, that the effect of the additional terms in (42a) is to increase slightly both the resistive and reactive portions of the effective surface impedance.

#### E. Numerical Determination of Effective Surface Impedance for Two Ocean Wind-Wave Models

##### 1. Background

The roughness present on the ocean surface is due almost entirely to the wind blowing across the water. Oceanographers have found that to excite ocean waves of a given spatial period,  $L$ , a wind with velocity greater than  $\sqrt{\frac{gL}{2\pi}}$  must blow for several hours ( $g$  is the acceleration of gravity). If the wind does not reach this given speed, then waves of this length are simply not present. If the wind speed does exceed this speed, the heights of the waves of period  $L$  remain constant, relatively unaffected by any further wind increase. Hence the shorter ocean waves are the first to be excited by an increasing wind, but after aroused, the height of these short waves remains fixed. In this sense the ocean wave-height spectrum is said to saturate with wind speed.

The saturated height vs spatial period for ocean waves has been the subject of experimental and theoretical studies to oceanographers for many years. Two recent monographs on ocean waves provide an excellent discussion and review of experimental evidence on this subject (Kinsman<sup>35</sup> and Phillips<sup>47</sup>). Over the past 15 years, several empirical models have been proposed, and two of these have found relatively wide

acceptance'. They differ mainly in the "power law" followed at saturation and in the shape of the long-wave cutoff vs wind speed. They are the Neumann-Pierson and the Phillips spectra.

It is not entirely correct to say that the ocean-wave heights at a given point on the sea are due solely to the winds that have blown across this point. Waves generated in a given area can and do propagate to portions of the ocean which have had considerably different wind histories. This is especially true of the longer, faster moving ocean waves. The roughness in a given area due to winds in other areas is known as swell. The ocean-wave spectra models to be discussed below neglect swell as a source of roughness, and relate the ocean-wave heights to the winds above that surface area. This oversimplification should be kept in mind before placing too much confidence in the wind-dependence of the effective surface impedance.

## 2. The Neumann-Pierson Spectrum

Neumann and Pierson first suggested a model which seemed to fit wavemeter measurements about 15 years ago. Their model was an isotropic\* temporal spectrum in which the saturated value is related to the temporal radian wave frequency as  $\omega^{-6}$ . To this they added a "guesstimated" lower end continuous cutoff function, since it seemed unreasonable to them that the cutoff should be abrupt. The resulting empirical law can be converted to an isotropic\* spatial spectrum. Kinsman<sup>35</sup> proposes a reasonable extension of this to produce a directional\* spatial spectrum with a cosine-squared dependence about the wind direction in azimuth. The spatial spectrum then has the form

$$W(p,q) = \frac{C(p \cos \alpha + q \sin \alpha)^2}{g^{5/2} (p^2 + q^2)^{13/4}} \exp\{-2g/(U^2 \sqrt{p^2 + q^2})\} \quad , \quad (44)$$

where  $U$  is the wind speed,  $g$  is the acceleration of gravity ( $9.81 \text{ m/s}^2$ ), and  $C$  is a constant empirically estimated to be  $C \approx 3.05 \text{ m}^2/\text{s}^5$ . The wind is assumed to be blowing in a direction  $\alpha$  with respect to the  $x$ -axis, and hence the main ocean waves move along this direction. The spectrum above is assumed non-zero by Kinsman only in the

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\*Isotropic means that ocean wave direction is not recorded and/or present. Some directionality would be expected, with dominant ocean waves moving in the wind direction. Unfortunately, too few measurements have been made to make a strong case for any given directional dependence.

half-space into which the wind is blowing; this is reasonable, for one would expect nearly all of the waves to be driven in the wind direction. The mean-square height of the ocean waves from the spectrum is  $\sigma^2 = \frac{3}{2} C (\pi/2)^{3/2} \cdot (U/2g)^5$ .

We now employ (44) in Equation (41) and evaluate the integral numerically for the two extremes in wind (and wave) direction:  $\alpha = 0$ , representing radio propagation along the dominant water-wave motion, and  $\alpha = \pi/2$  representing propagation across the dominant water wave direction. We call these two cases the upwind-downwind and crosswind directions respectively. In the integration, we divide (44) by two and assume the spectrum exists over all space instead of over only the forward wind half. This is necessary because at a given instant, the sea profile will appear "frozen" to a radio wave, and it will not be possible to tell from this profile whether the waves are moving forward or backward.

Figures 4 and 5 show the curves of the resistive (solid lines) and reactive (dashed lines) portions of the normalized effective surface impedance,  $\bar{\Delta}$ . The sea water is assumed to have a conductivity of 4 mhos/meter, which is the average between the winter and summer values in the Atlantic. The curves indicate that the most severe changes in surface impedance occur for a radio wave propagating in the direction of the dominant ocean waves (i.e., across the corrugations), as one would expect. These curves are really only valid out to frequencies and wind speeds such that  $k_0^2 \sigma^2 \ll 1$ ,\* since this is one of the assumptions of the analysis. For example, the curves could be used up to a frequency of about 30 MHz with a wind speed of 25 knots.

### 3. The Phillips Spectrum

Ocean-wave data gathered in recent years have led many oceanographers to search for an alternative to the Neumann-Pierson spectrum. Phillips<sup>47</sup> and Munk<sup>48</sup> have presented convincing evidence to show that the spatial spectrum should behave as  $(p^2 + q^2)^{-2}$ , instead of the  $(p^2 + q^2)^{-5/4}$  dependence of Equation (44) for large  $p^2 + q^2$ . More important, however, measurements show that the lower-end cutoff is much more pronounced than the artificial exponential factor attached to the Neumann-Pierson model in (44). Thirdly, slope measurements by Cox and Munk<sup>49</sup> show that the spectrum is much closer to being isotropic than the cosine-squared directionality of (44). Hence, they suggest the following spectrum<sup>47</sup>:

$$*k_0^2 \sigma^2 = \frac{3}{2} C k_0^2 (\pi/2)^{3/2} (U/2g)^5$$

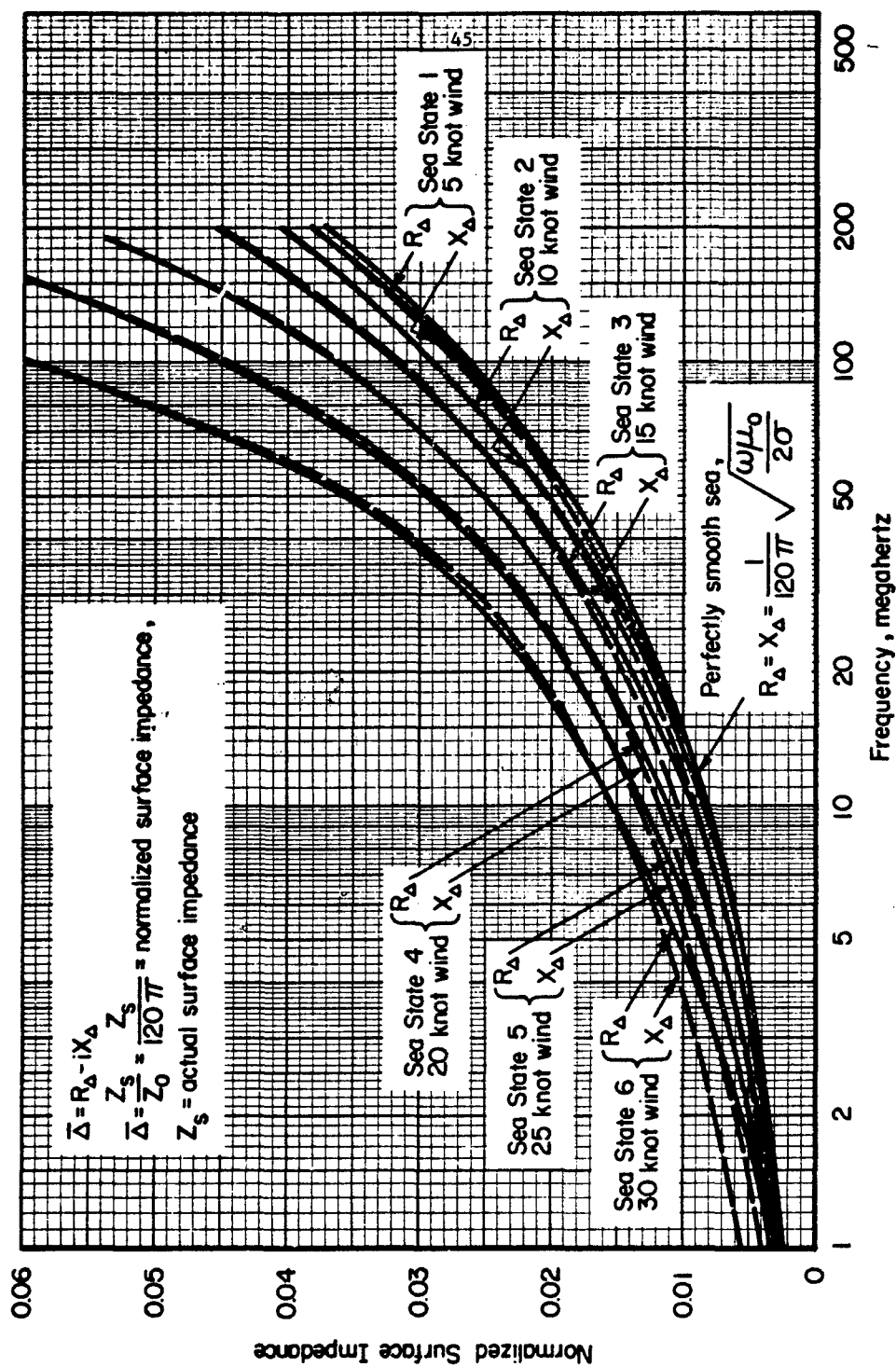


Figure 4. Effective Surface Impedance,  $\bar{\Delta}$ , vs Wind Speed and Frequency. Neumann-Pierson Ocean-Wave Spectrum and Propagation in Upwind-Downwind Direction.



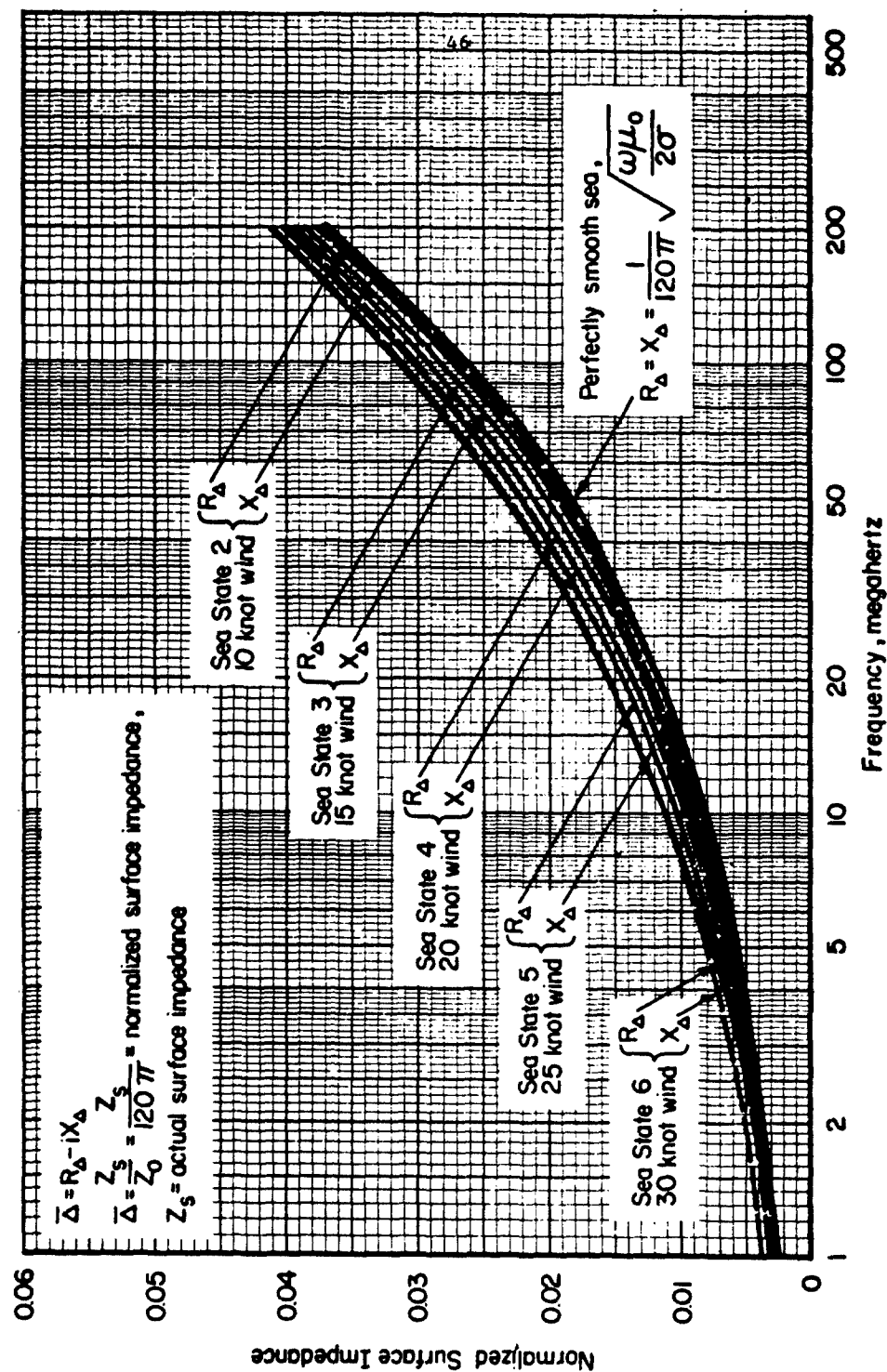


Figure 5. Effective Surface Impedance,  $\bar{\Delta}$ , vs Wind Speed and Frequency. Neumann-Pierson Ocean-Wave Spectrum and Propagation in Crosswind Direction.

$$W(p,q) = \frac{4B}{\pi(p^2 + q^2)^2} \quad (45)$$

where  $B \approx 0.005$ . The spectrum is identically zero for  $\sqrt{p^2 + q^2} < g/U^2$  and also in the half-space from which the wind is coming. The cutoff exhibited by (45) is much sharper than that of (44).

We use (45) in a numerical evaluation of (41), and the results are shown in Figure 6. As before, we divide (45) by two since the "frozen" surface profile must produce a symmetric spectrum about the origin. For (45), the mean-square height is  $\sigma^2 = \frac{1}{2} B U^4 / g^2$ . Since the curves are meaningful only for  $k_0^2 \sigma^2 \ll 1$ , they can be used for frequencies up to about 40 MHz with a wind speed of 25 knots. Again, the conductivity of the sea is taken as 4 mhos/meter.

In comparison, the upwind-downwind impedances from the Neumann-Pierson spectrum are the largest. The impedances from the Phillips spectrum are reasonably close to those for the upwind-downwind direction with the Neumann-Pierson spectrum. Those for the crosswind direction of the Neumann-Pierson spectrum are the smallest, as expected. Until experimental evidence is obtained, we believe that the Phillip's impedances are perhaps the best estimate of the three.

#### F. Basic Transmission Loss Calculations

In the spirit of Wait's analyses<sup>34</sup>, we can use the effective (or equivalent) impedance,  $\bar{\Delta}$ , in the integrals for ground-wave propagation. To facilitate the calculation, Berry and Chrisman of ESSA have prepared a computer program<sup>36</sup> which solves this integral for the field strength above a spherical earth. The program selects one of three techniques for solving the integral, depending on whether the observation point is clearly visible to the source, in the penumbra, or in the deep shadow. In the visible region, a saddle-point evaluation is employed. In the penumbra, the computer does a complex numerical integration around a contour enclosing the poles of the integrand. In the deep shadow region, the residue series is used; it involves Fock's forms of the Airy functions and their roots rather than the Hankel functions of one-third order, as used by Bremmer and van der Pol. The computer output has been checked against Norton's curves and also against the known solutions in the visible region, and the results agree.

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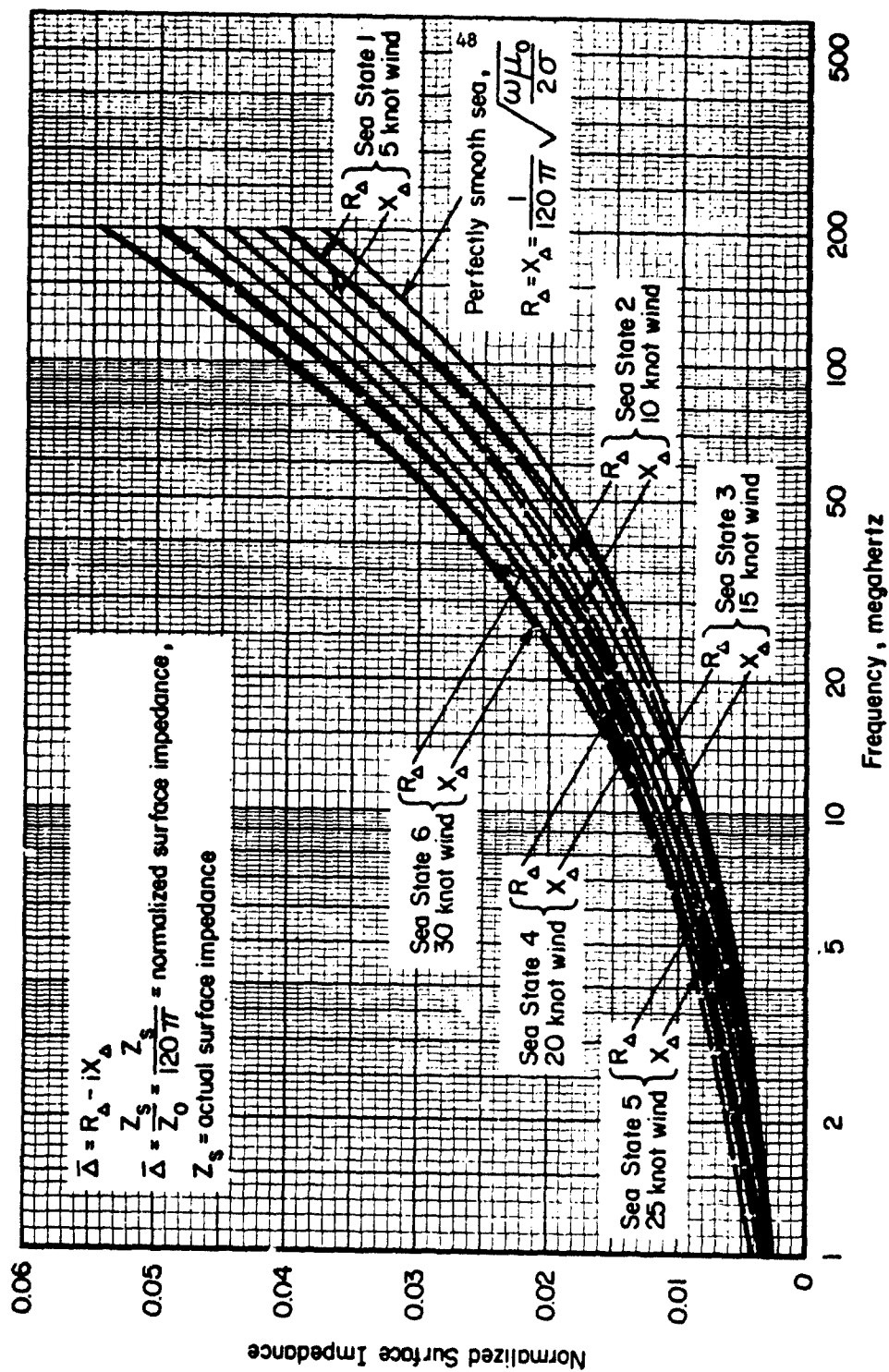


Figure 6. Effective Surface Impedance,  $\bar{\Delta}$ , vs Wind Speed and Frequency.  
 Phillips Isotropic Ocean-Wave System.

We employ this computer program to account for sea-state effects with the following minor modifications: (i) we alter the input format so that surface impedance,  $\bar{\Delta}$ , can be used rather than conductivity and dielectric constant, and (ii) we add another variable to the output, basic transmission loss,  $L_b$ .

Basic transmission loss is a concept widely publicized by Norton<sup>50,51</sup> in the 1950's. Formally, it is defined as

$$L_b = 10 \log_{10} \left( \frac{P_{ti}}{P_{ri}} \right) \quad (46)$$

where  $P_{ti}$  is the power transmitted by an isotropic radiator and  $P_{ri}$  is the power received by an isotropic radiator. In a simple communication problem, one must merely subtract out the free-space antenna gains (in dB) in order to determine the overall power loss. For example, the basic transmission loss between two points in free space separated by distance  $d$  is  $10 \log_{10} (4\pi d/\lambda)^2$ ; if the same two points are located above a flat, perfectly conducting ground plane, the 4 is replaced by 2 in the parentheses. The presentation of ground-wave attenuation in the form of basic transmission loss appears to be more readily interpretable and useful than many of the earlier ground-wave normalizations (e.g., unity electric dipole, 1 kW transmitted power, field strength one mile from the transmitter, etc.). Examples illustrating the application of these basic transmission loss curves will be provided in the next section.

For reference, we show curves of basic transmission loss just above a perfectly smooth, perfectly conducting planar surface in Figure 7, as a function of range. The values for loss shown are essentially 6 dB lower than the loss through free space because the vertically polarized field radiated by an isotropic source on the surface is twice as large with the plane present. Figure 8 shows the basic transmission loss over a perfectly smooth spherical sea vs range. These curves were computed from the ESSA ground-wave program, using  $\epsilon_r = 80$ ,  $\sigma = 4$  mhos/meter, and effective earth radius = 4/3 actual earth radius. The former conductivity is typical of sea water in the Atlantic ocean. The curves are calculated for frequencies spanning the HF and lower VHF regions. At short ranges, the curves of Figure 8 coincide with those of Figure 7, as they should.

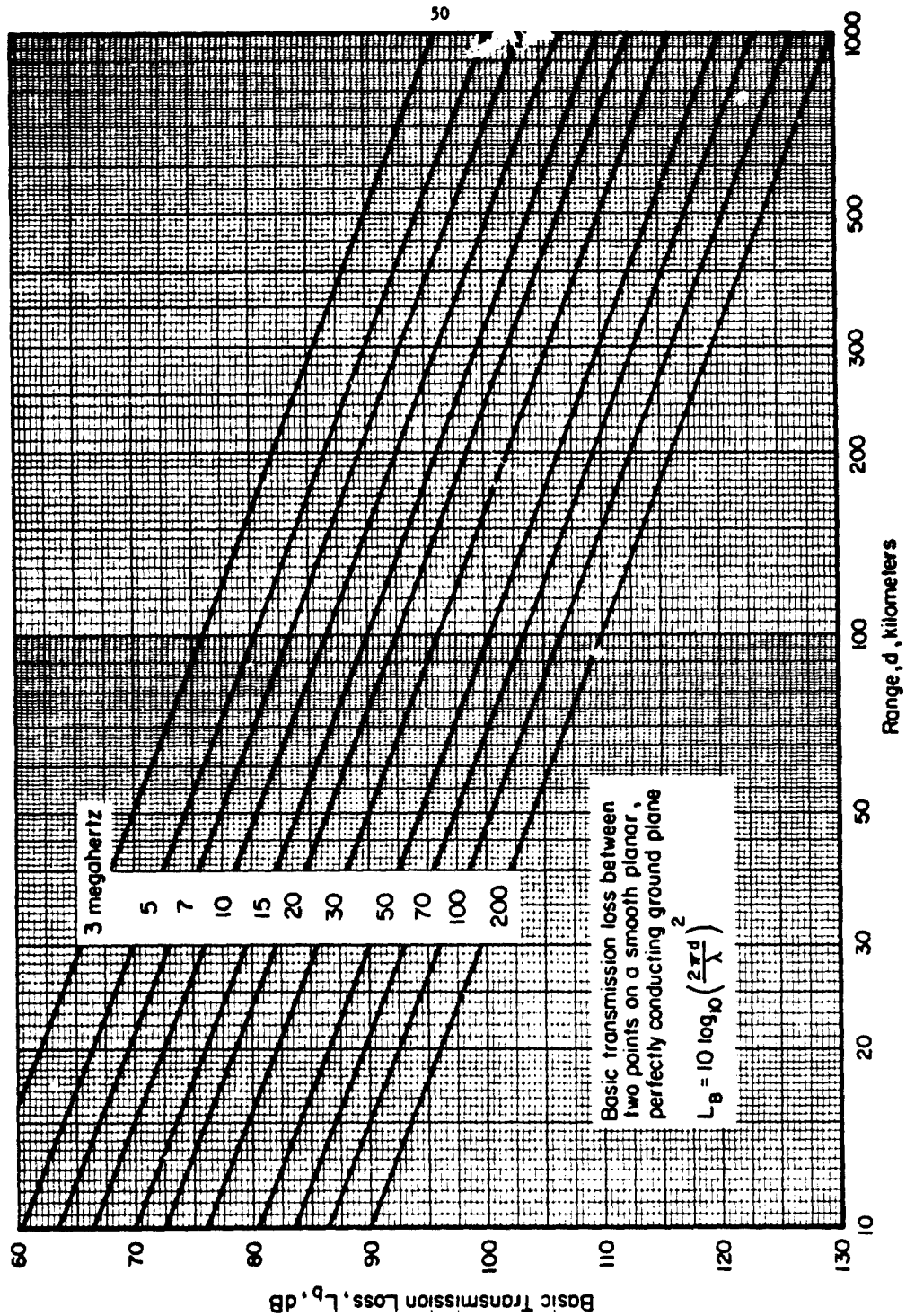


Figure 7. Basic Transmission Loss Across a Flat, Perfectly Conducting, Perfectly Smooth Ground Plane vs Range and Frequency. Transmitter and Receiver Antennas are at the Surface.

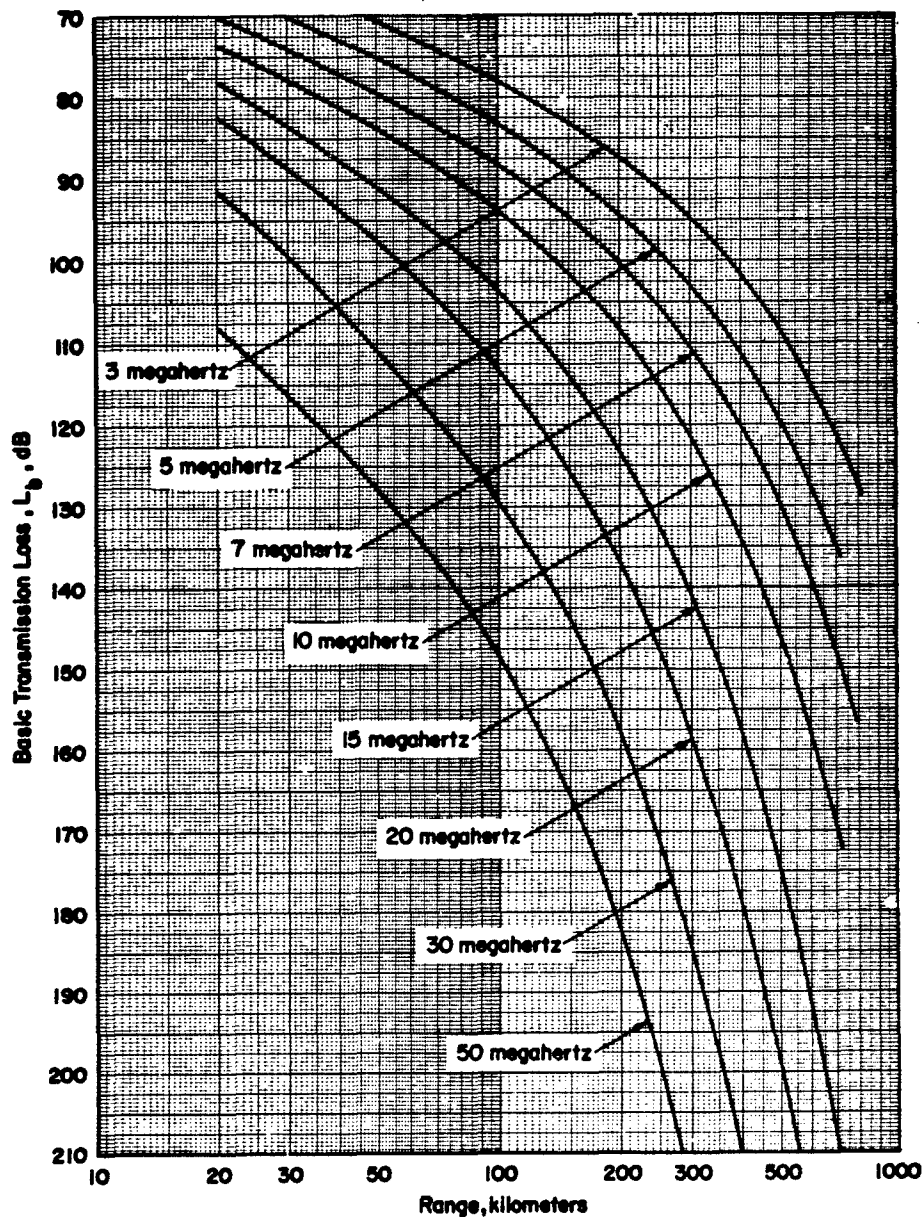


Figure 8. Basic Transmission Loss Across the Ocean Between Points at the Surface of Smooth Spherical Earth. Conductivity is 4 mhos/meter and an Effective Earth Radius Factor of  $4/3$  is Assumed.

To clearly show the effects of sea state and separate them from the normal attenuation experienced by a ground-wave above a smooth spherical sea, we compute the difference between  $L_b$  for the smooth sea, as shown in Figure 8 and  $L_b$  for the rough sea. The latter  $L_b$  for the rough sea is computed from the ESSA program using the values of  $\bar{A}$  (surface impedance) obtained earlier, accounting for sea state. The difference in transmission losses between the smooth and rough sea is termed  $L_{ss}$ , and shown in Figures 9 through 32. Again, the conductivity of the sea water is taken as 4 mhos/meter and the effective earth radius factor (accounting for refractivity) is 4/3. The source and receiver are both assumed to be located at the surface. Eight frequencies are selected (3, 5, 7, 10, 15, 20, 30, and 50 MHz) which span the HF and lower VHF region. Figures 9 through 16 represent the Neumann-Pierson ocean spectrum for propagation in the upwind-downwind direction; Figures 17 through 24 use the same spectrum, but for propagation in the crosswind direction. Finally Figures 25 through 32 present results for the isotropic version of the Phillips ocean spectrum.

Figures 33 through 42 show a profile of basic transmission loss to various points on and above the earth's surface. The source is assumed to be located at the surface. The first number at each grid point represents the transmission loss when the sea is perfectly smooth, while the second number represents the transmission loss for Sea State 5 (25 knot wind) using the Phillips spectrum.

## V. EXAMPLES OF THE USE OF THE GRAPHS

### A. Surface-to-Surface Communication Problem

A shore station with an array antenna is to communicate with a ship 300 km away on a frequency of 10 MHz. The shore-based array has an equivalent free-space\* gain of  $G_t = 17$  dB and the shipboard antenna has a free-space\* gain of  $G_r = 6$  dB when the two beams are optimally aligned. The shore station transmits an average power of 1 kW. We are to find the total power at the receiver terminals before amplification for (a), a smooth sea, and (b), a sea generated by a 25-knot wind.

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\*This means that the gains are measured or calculated as though the antenna were isolated in free space rather than being located over a highly conducting ground.

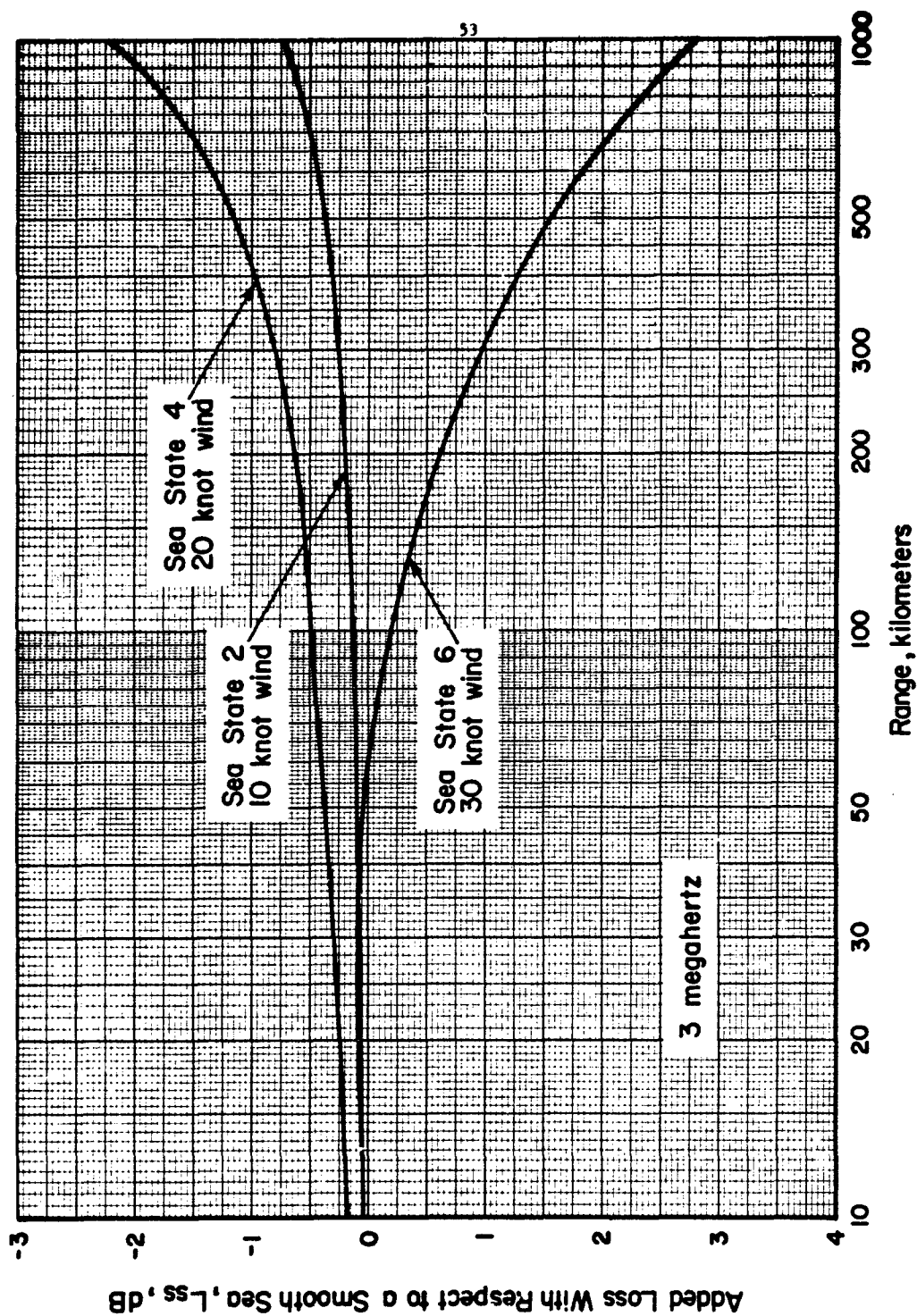


Figure 9. Added Transmission Loss Due to Sea State at 3 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.



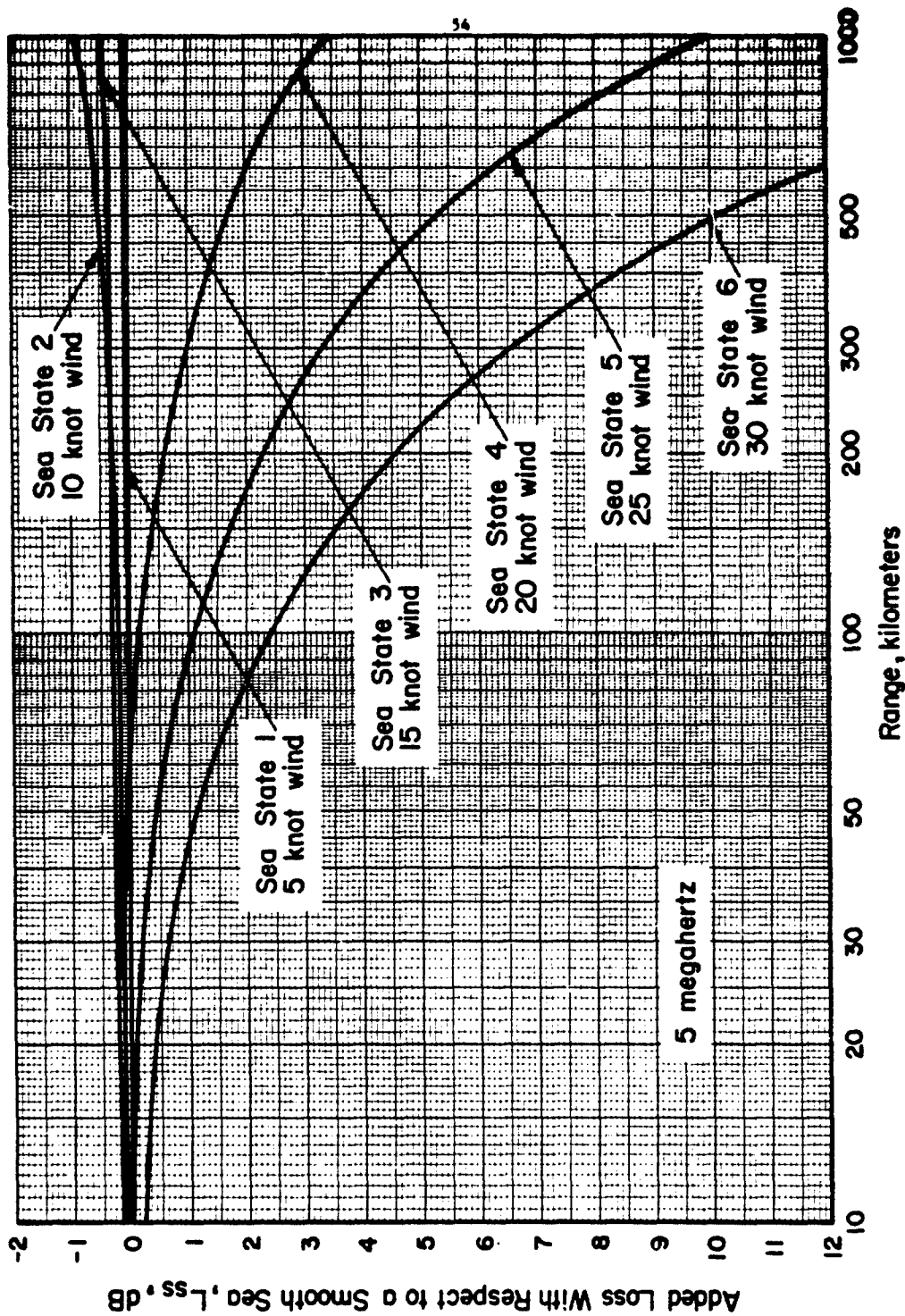


Figure 10. Added Transmission Loss Due to Sea State at 5 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

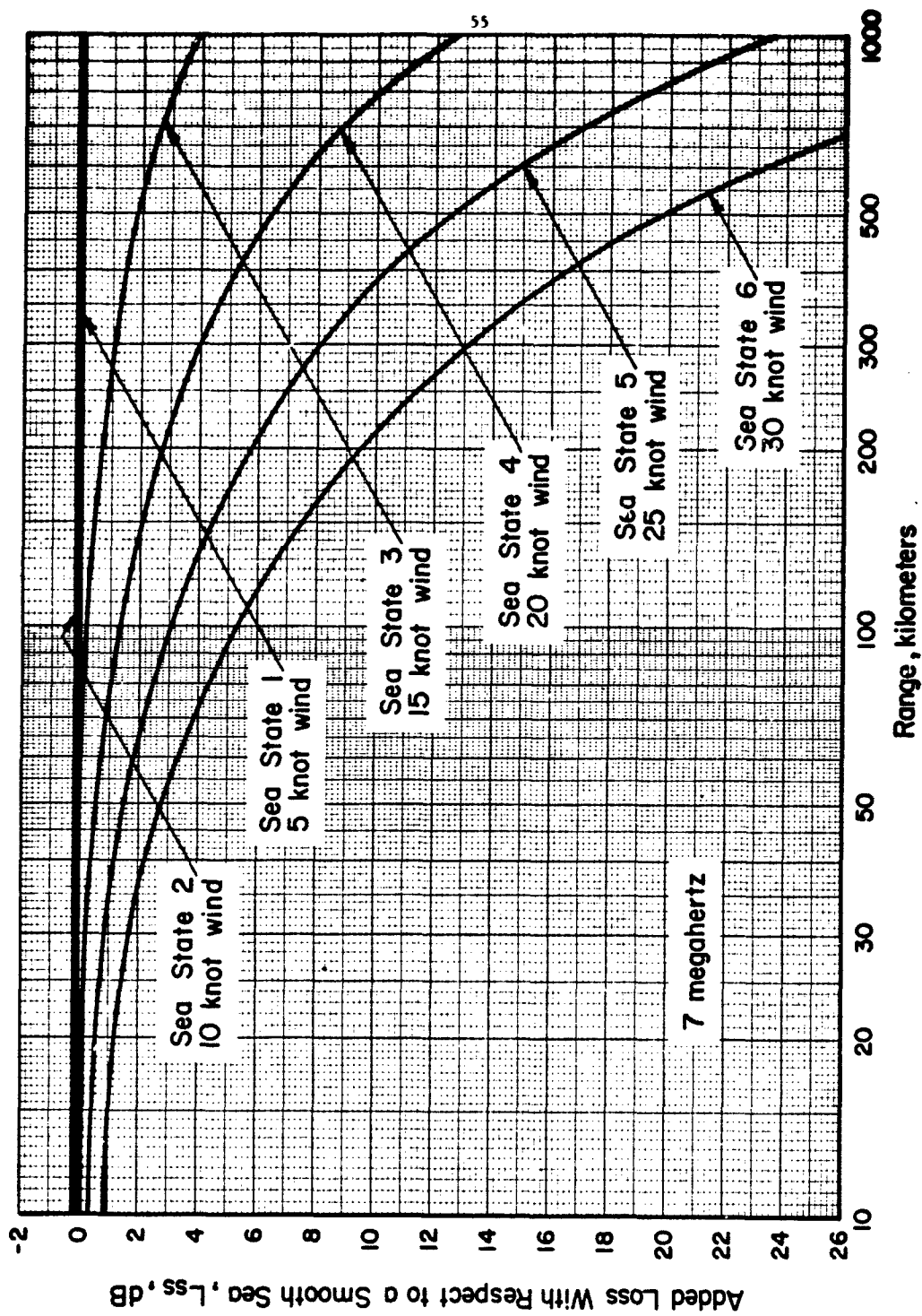


Figure 11. Added Transmission Loss Due to Sea State at 7 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

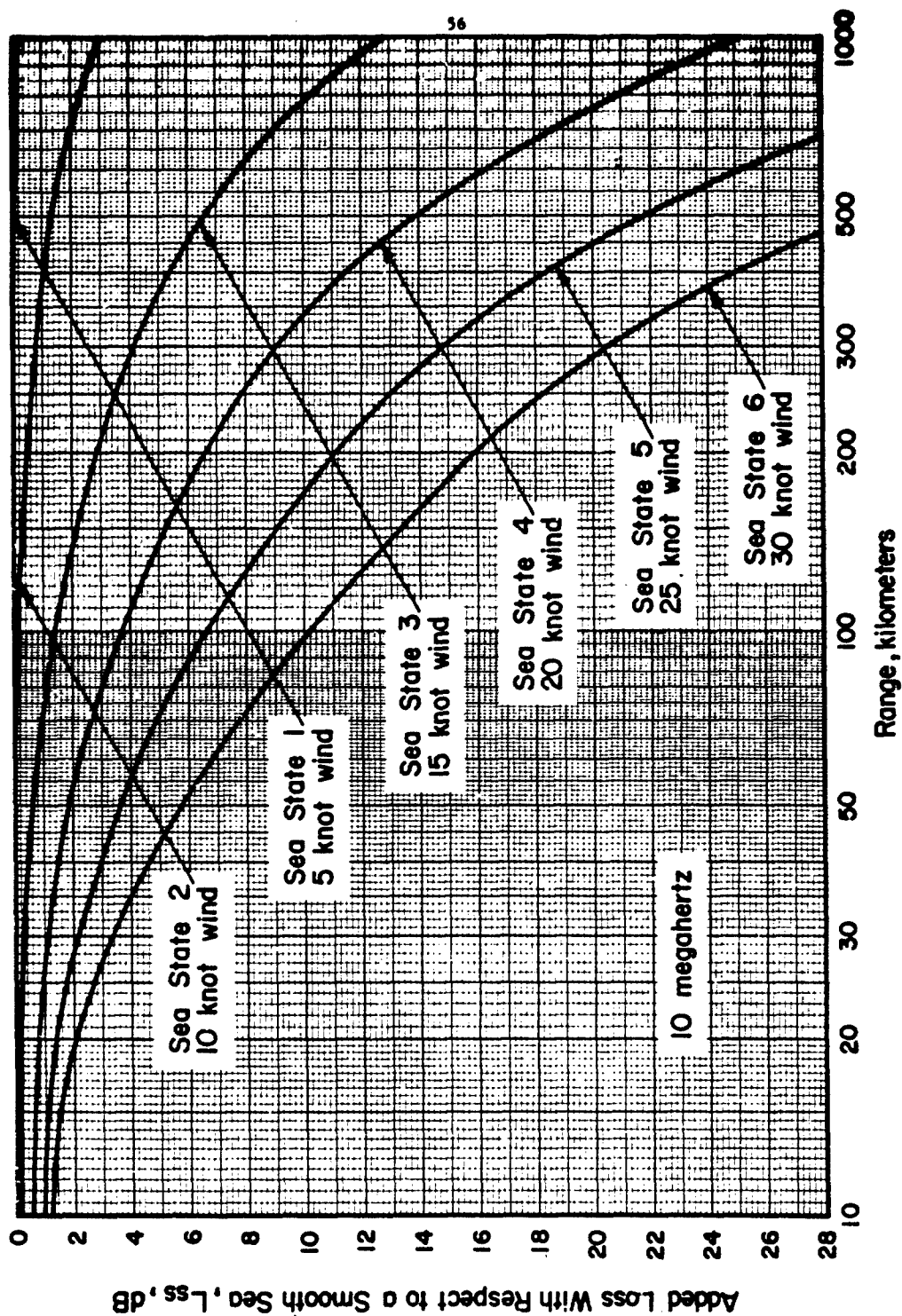


Figure 12. Added Transmission Loss Due to Sea State at 10 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

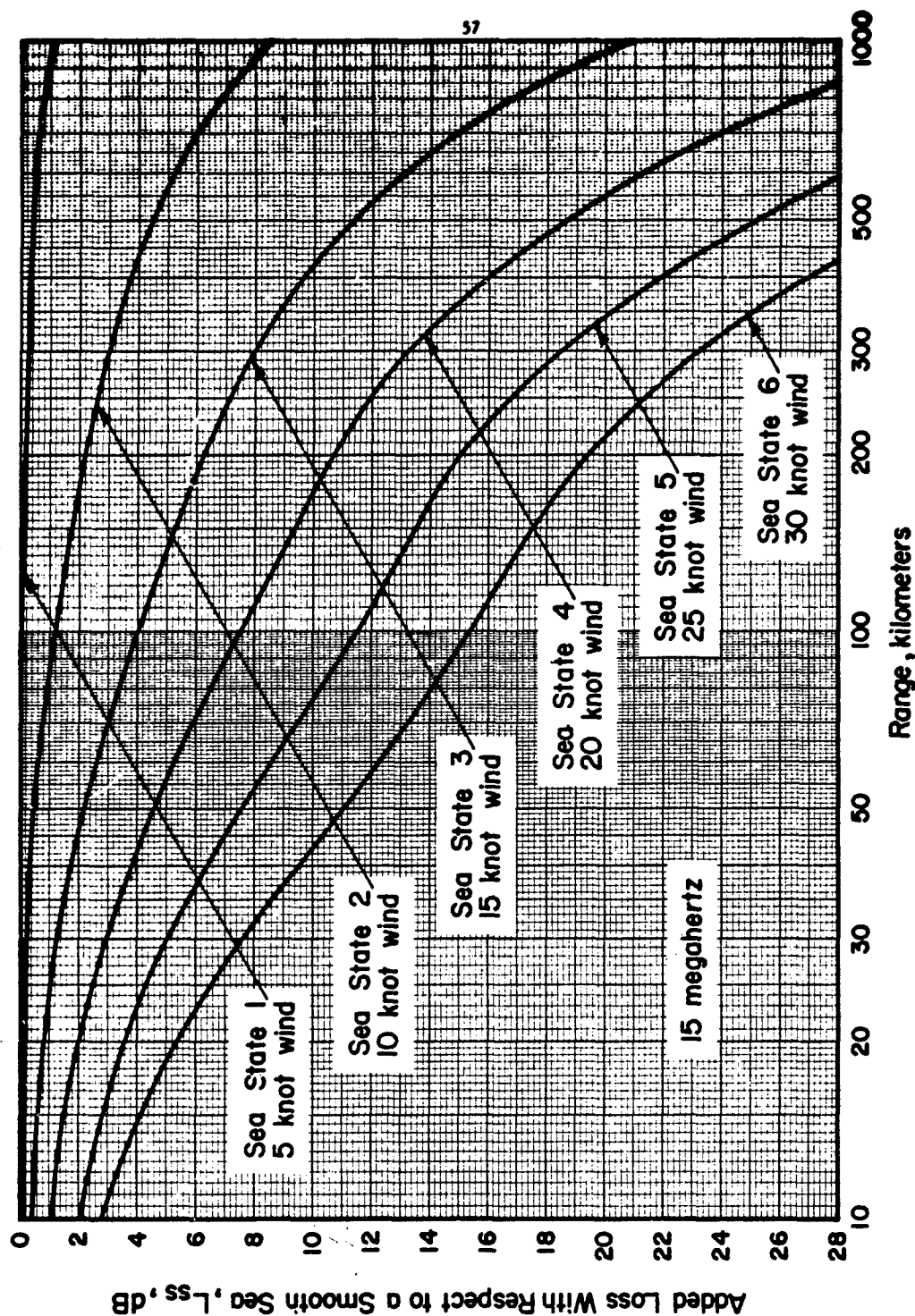


Figure 13. Added Transmission Loss Due to Sea State at 15 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

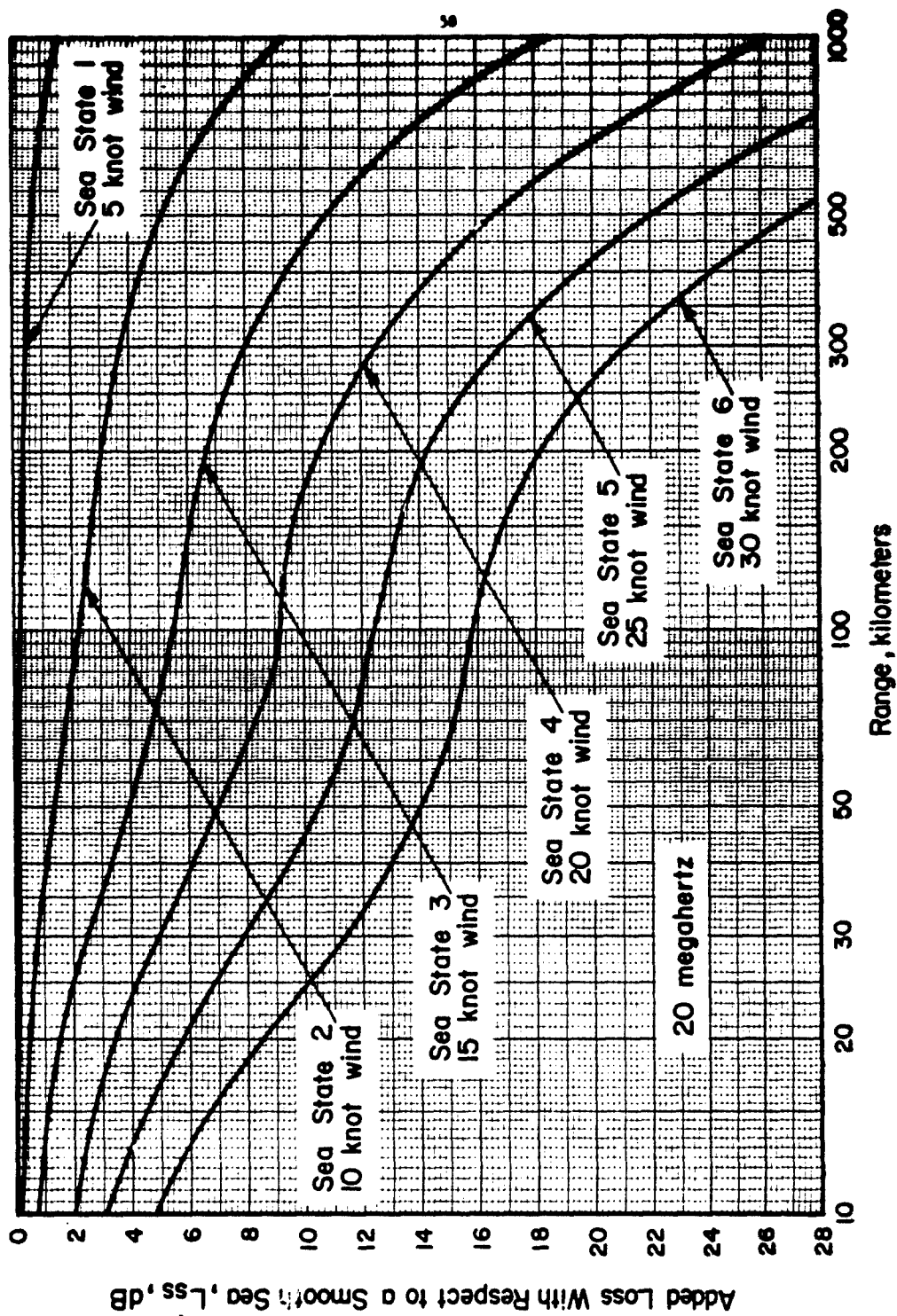


Figure 14. Added Transmission Loss Due to Sea State at 20 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

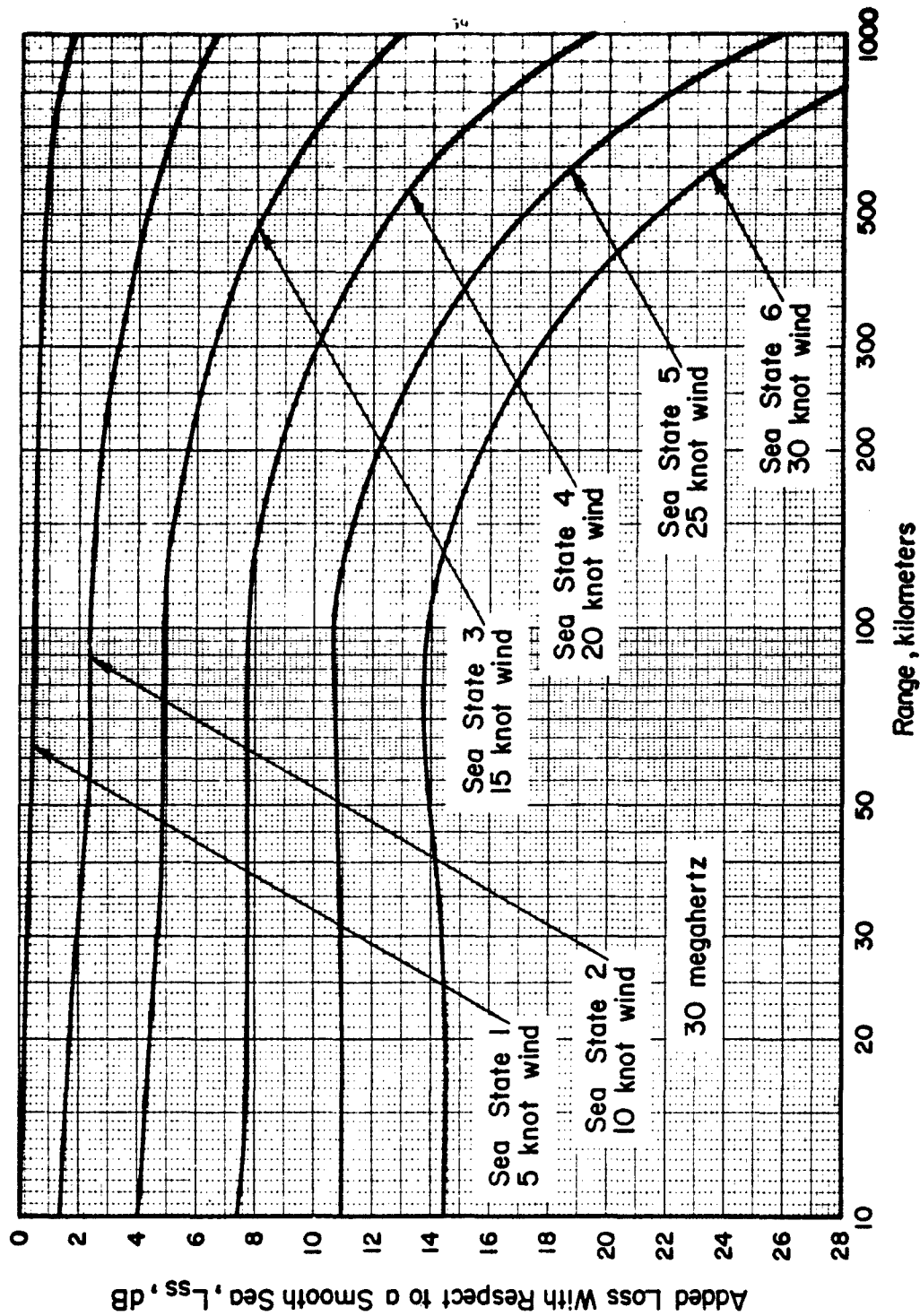


FIGURE 15. Added Transmission Loss Due to Sea State at 30 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.

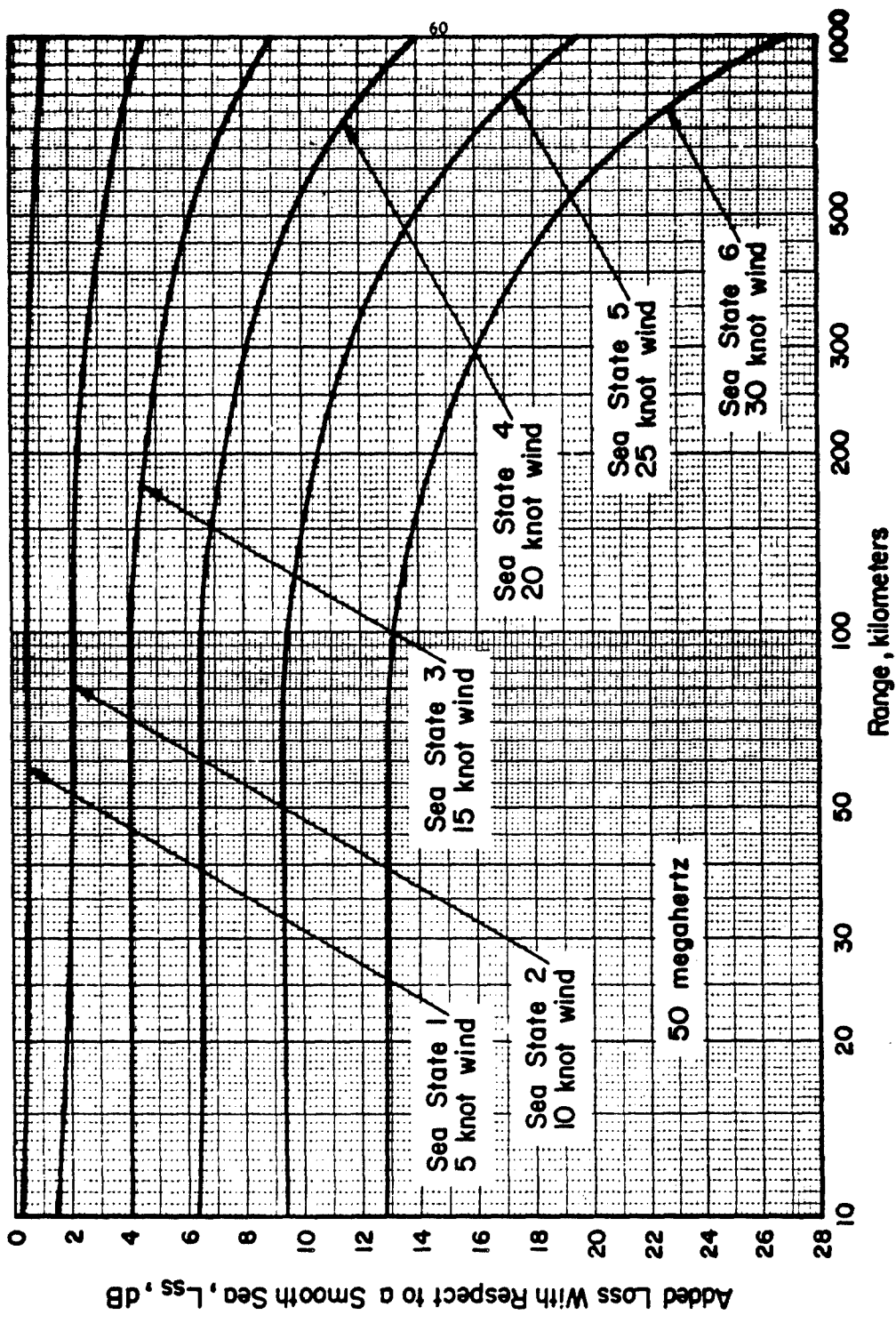


FIGURE 16. Added Transmission Loss Due to Sea State at 50 MHz. Antennas are Located Just Above Surface. New Jan-Pierson Ocean-Wave Spectrum with Propagation in Upwind-Downwind Direction.



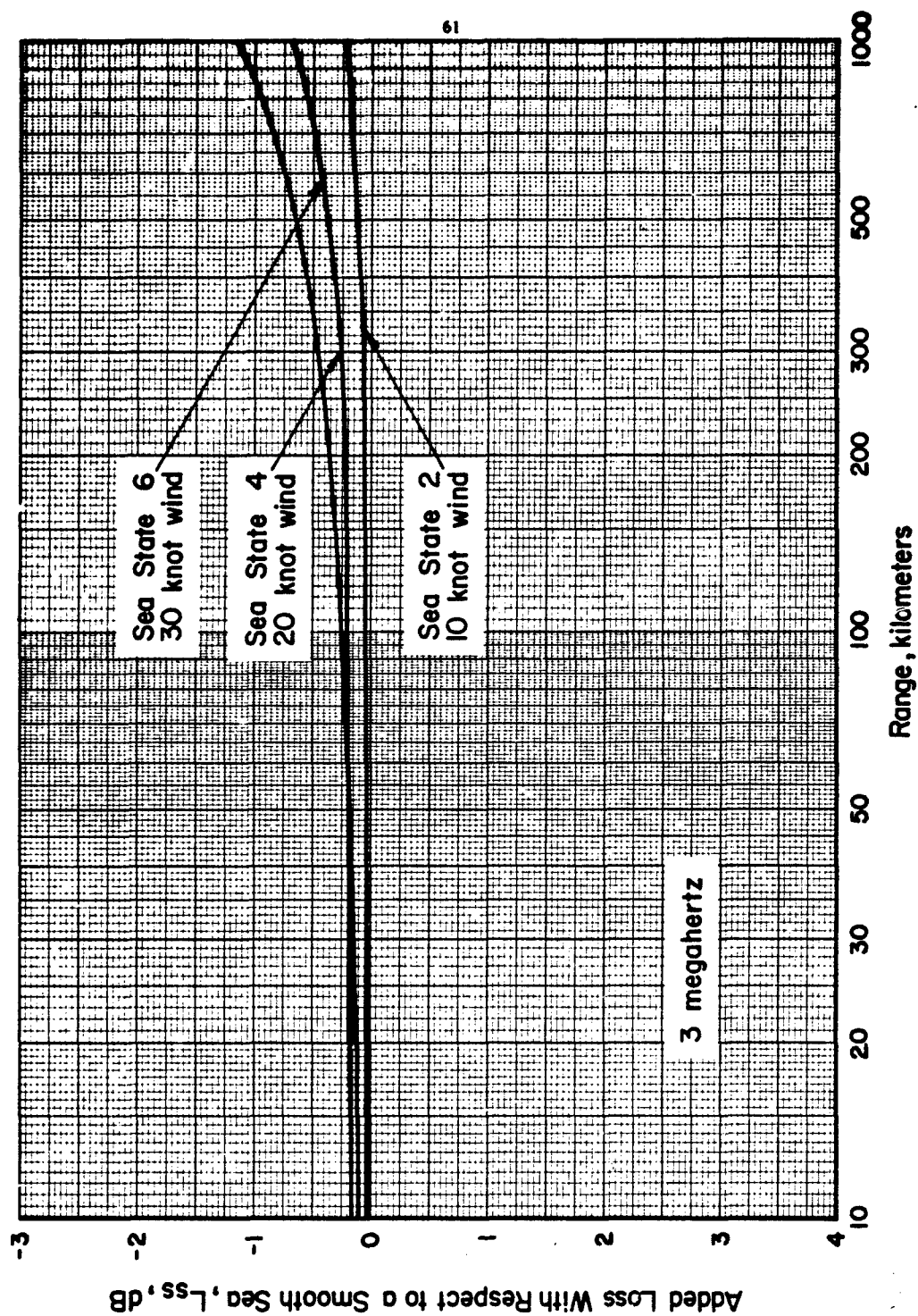


Figure 17. Added Transmission Loss Due to Sea State at 3 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.



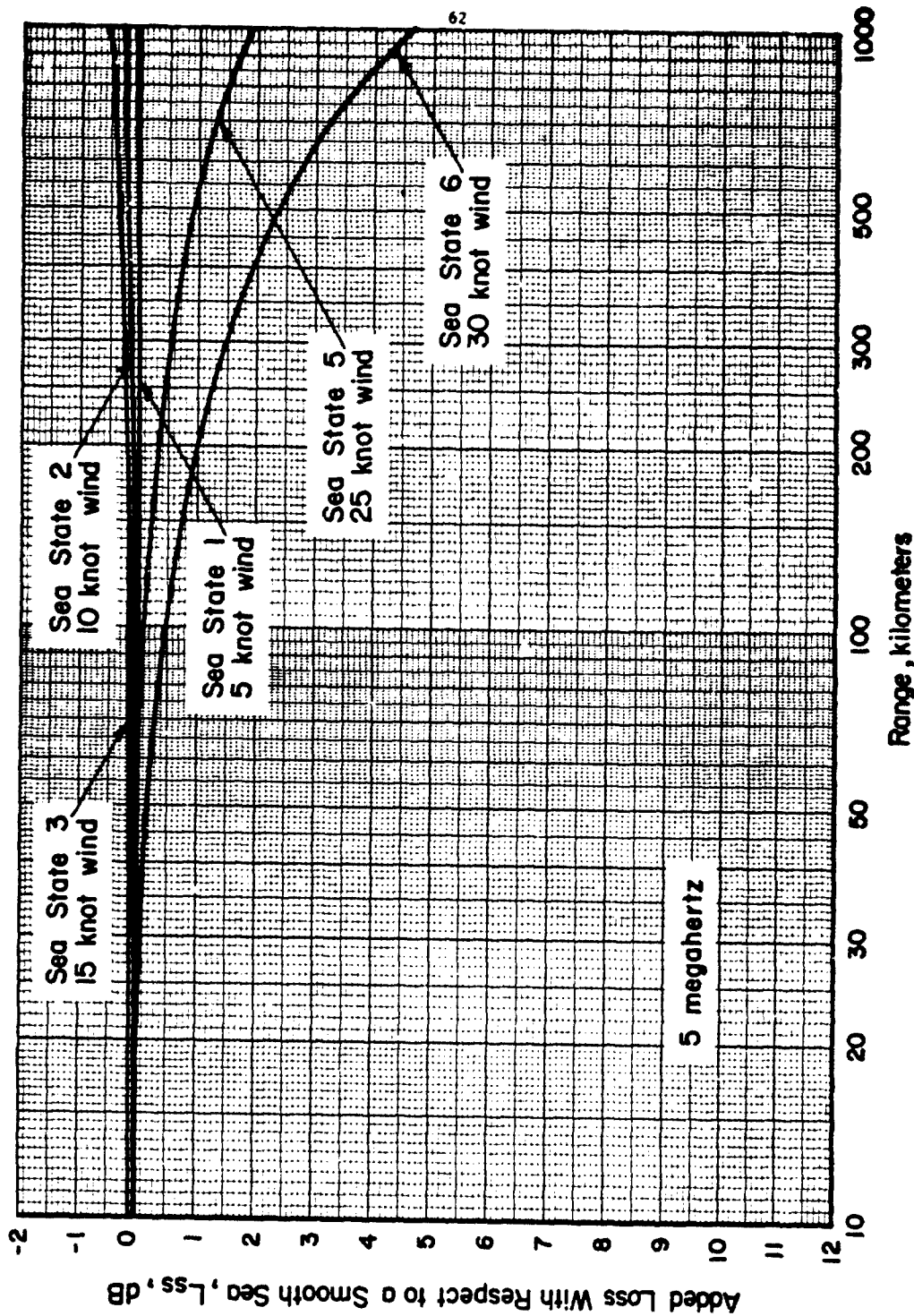


Figure 18. Added Transmission Loss Due to Sea State at 5 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

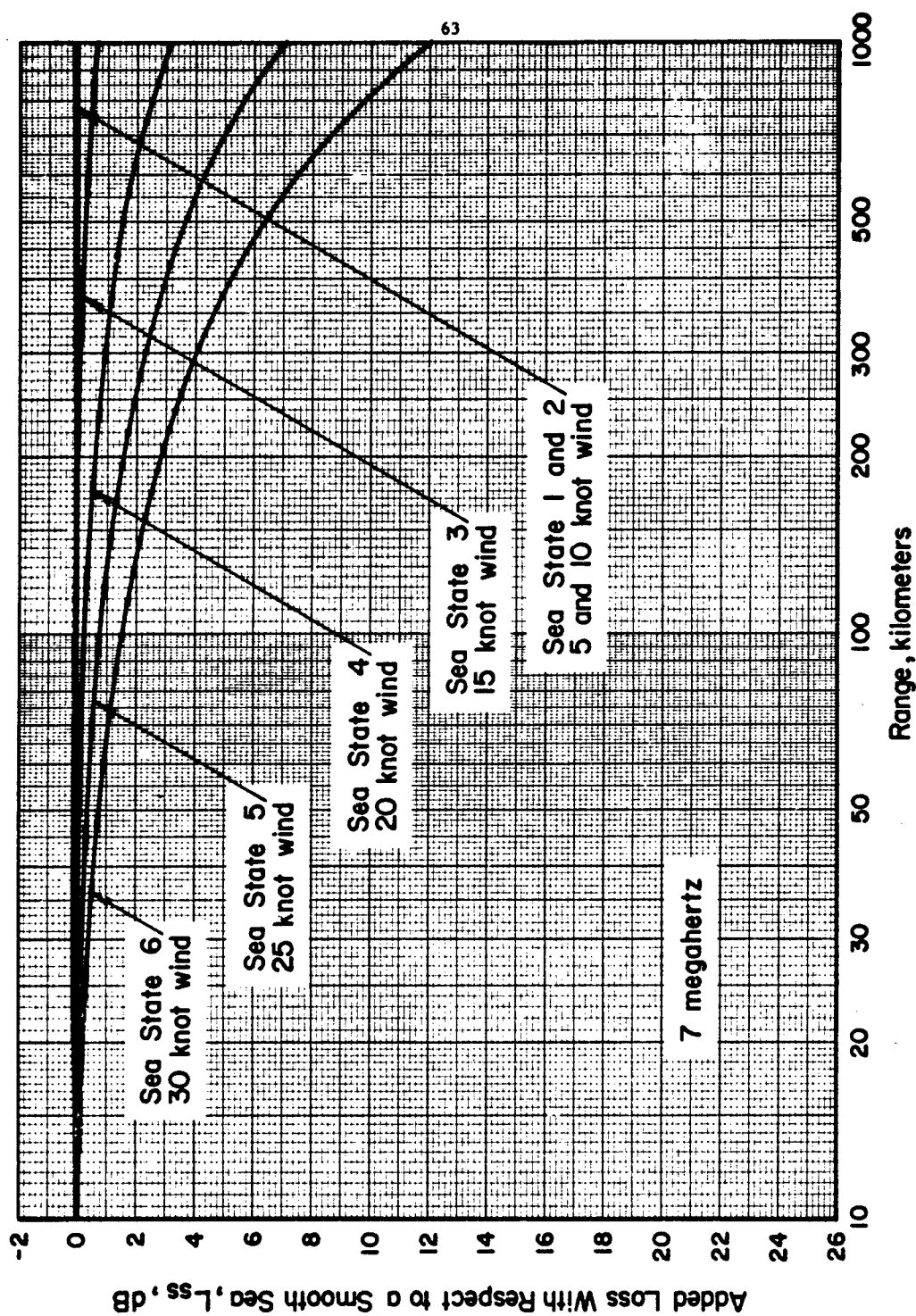


Figure 19. Added Transmission Loss Due to Sea State at 7 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

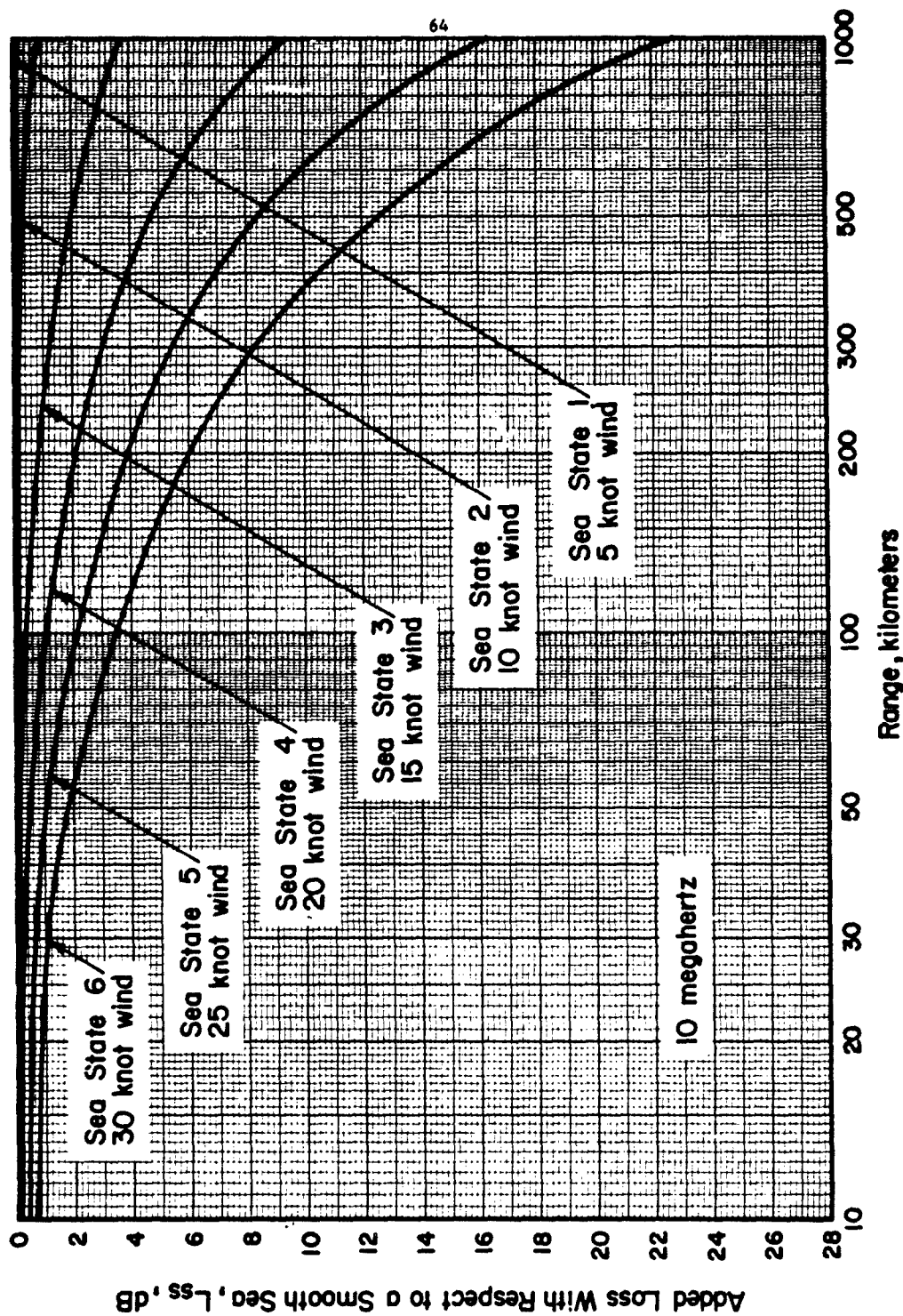


Figure 20. Added Transmission Loss Due to Sea State at 10 MHz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

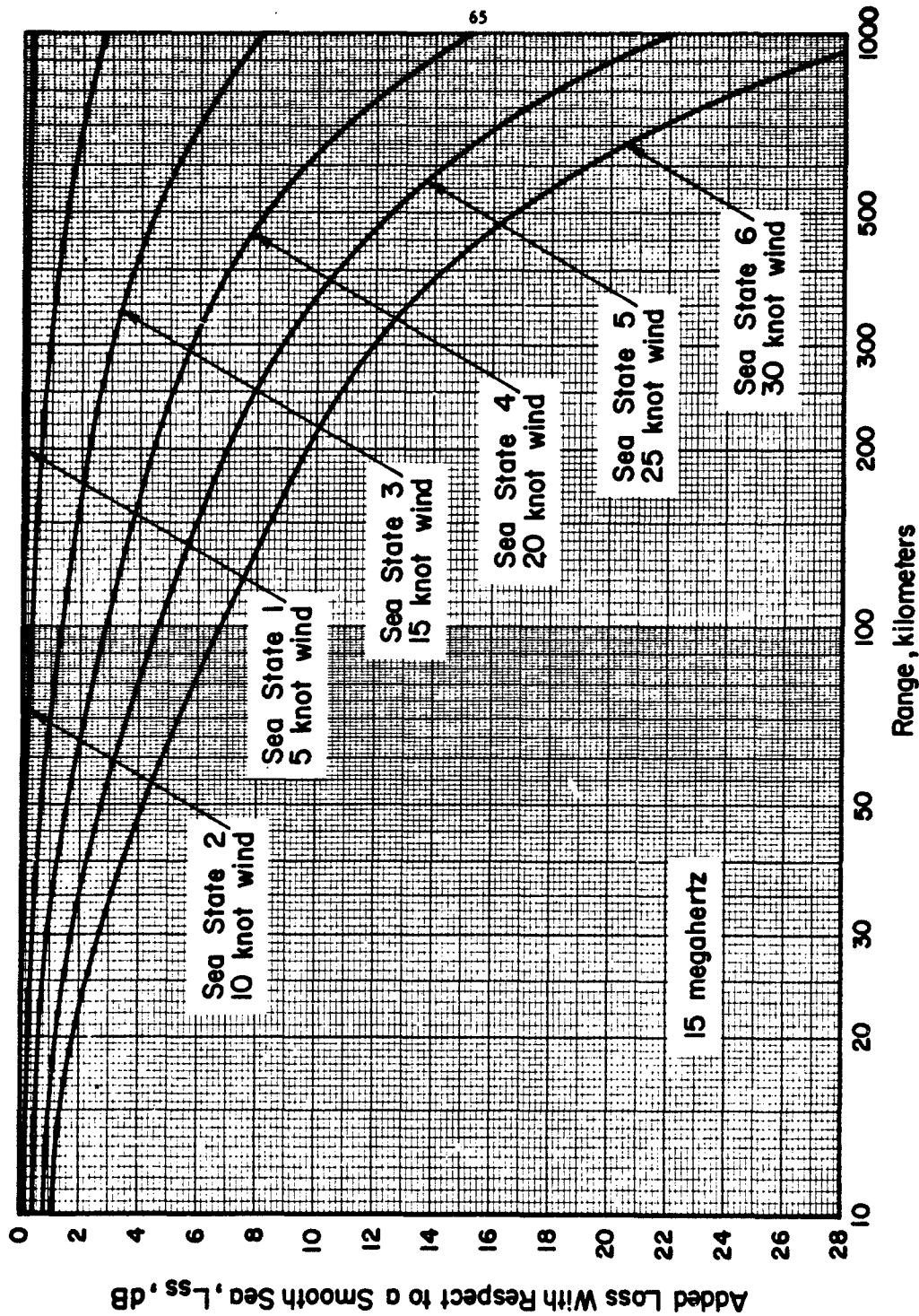


Figure 21. Added Transmission Loss Due to Sea State at 15 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

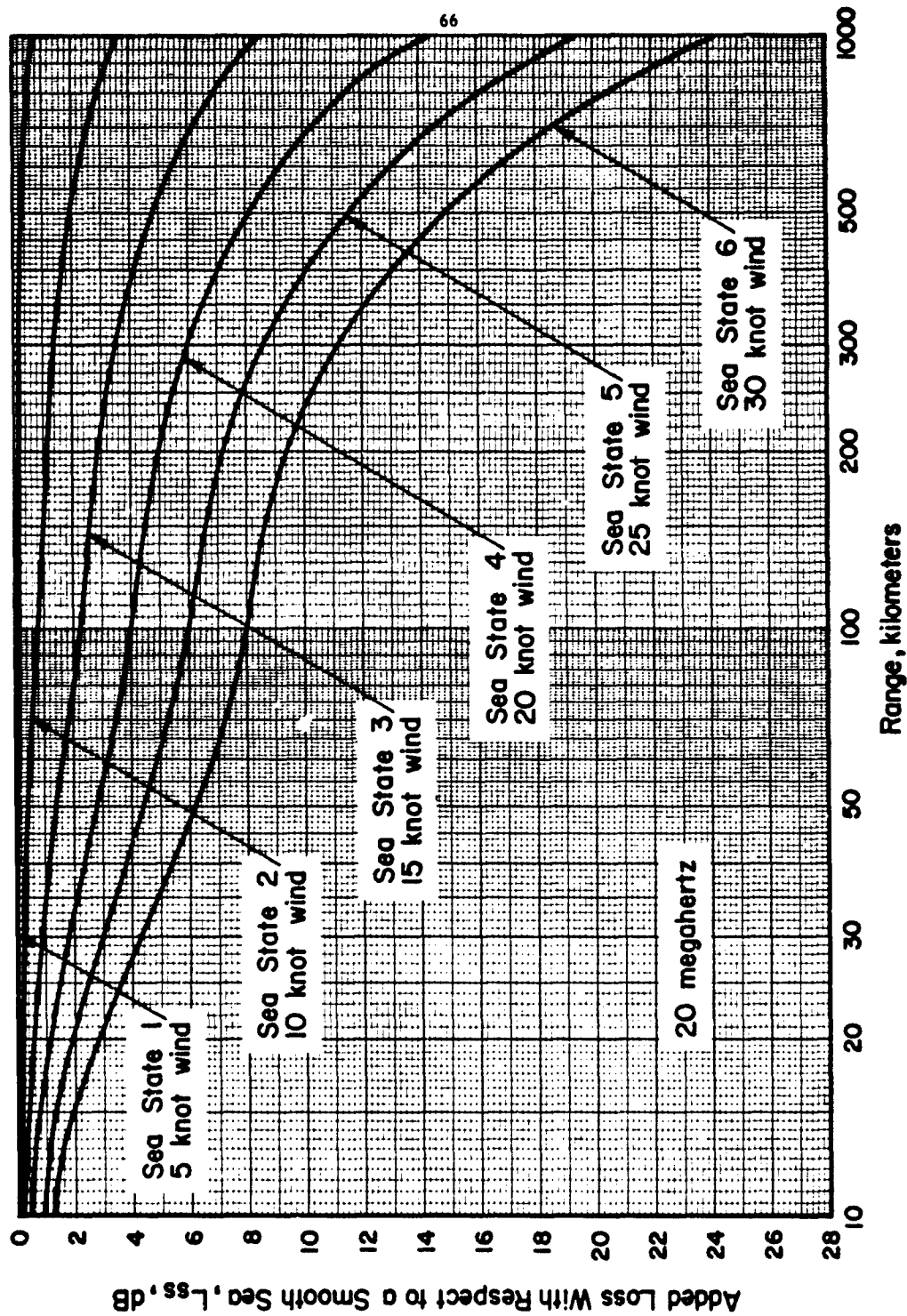


Figure 22. Added Transmission Loss Due to Sea State at 20 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

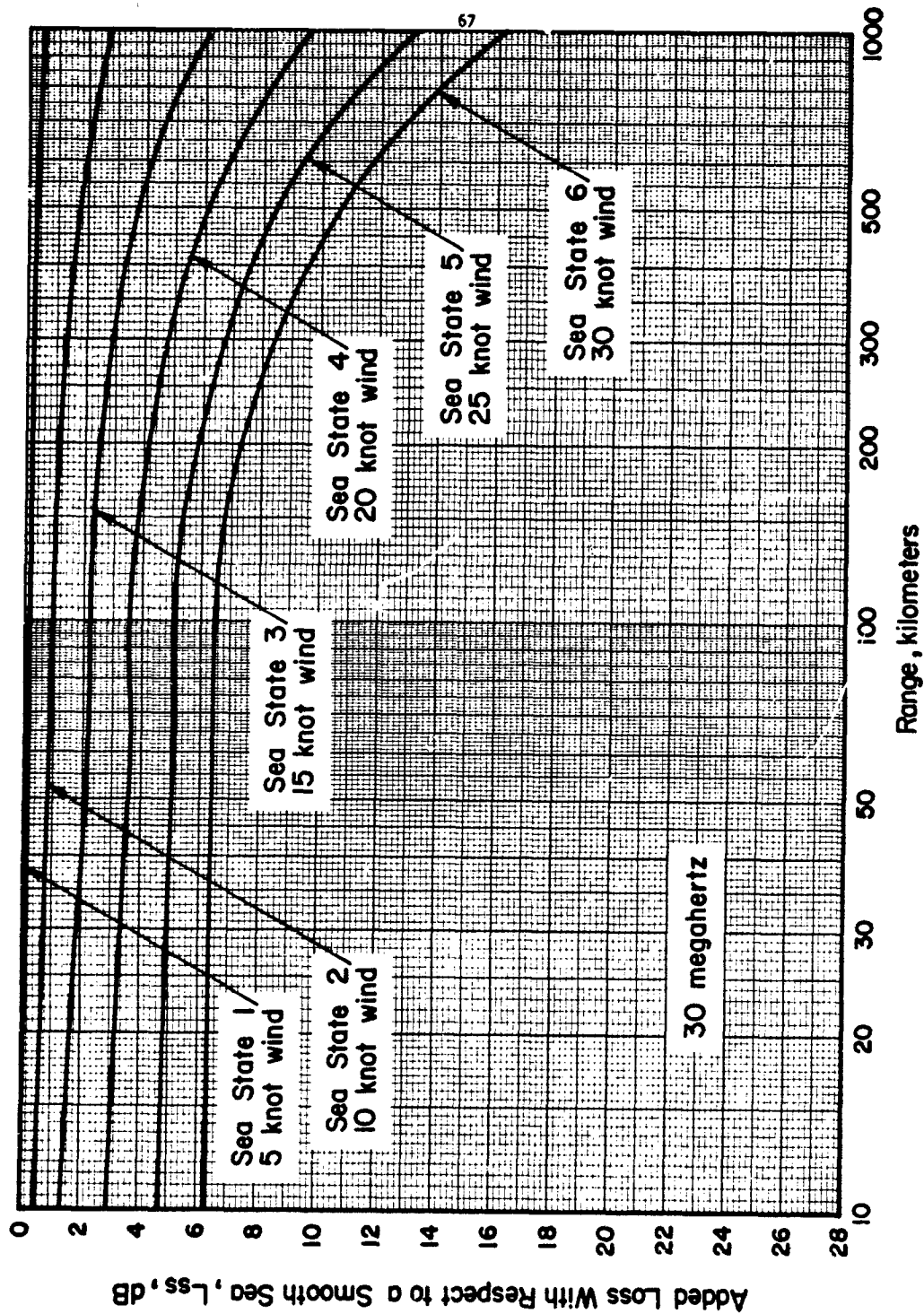


FIGURE 23. Added Transmission Loss Due to Sea State at 30 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.



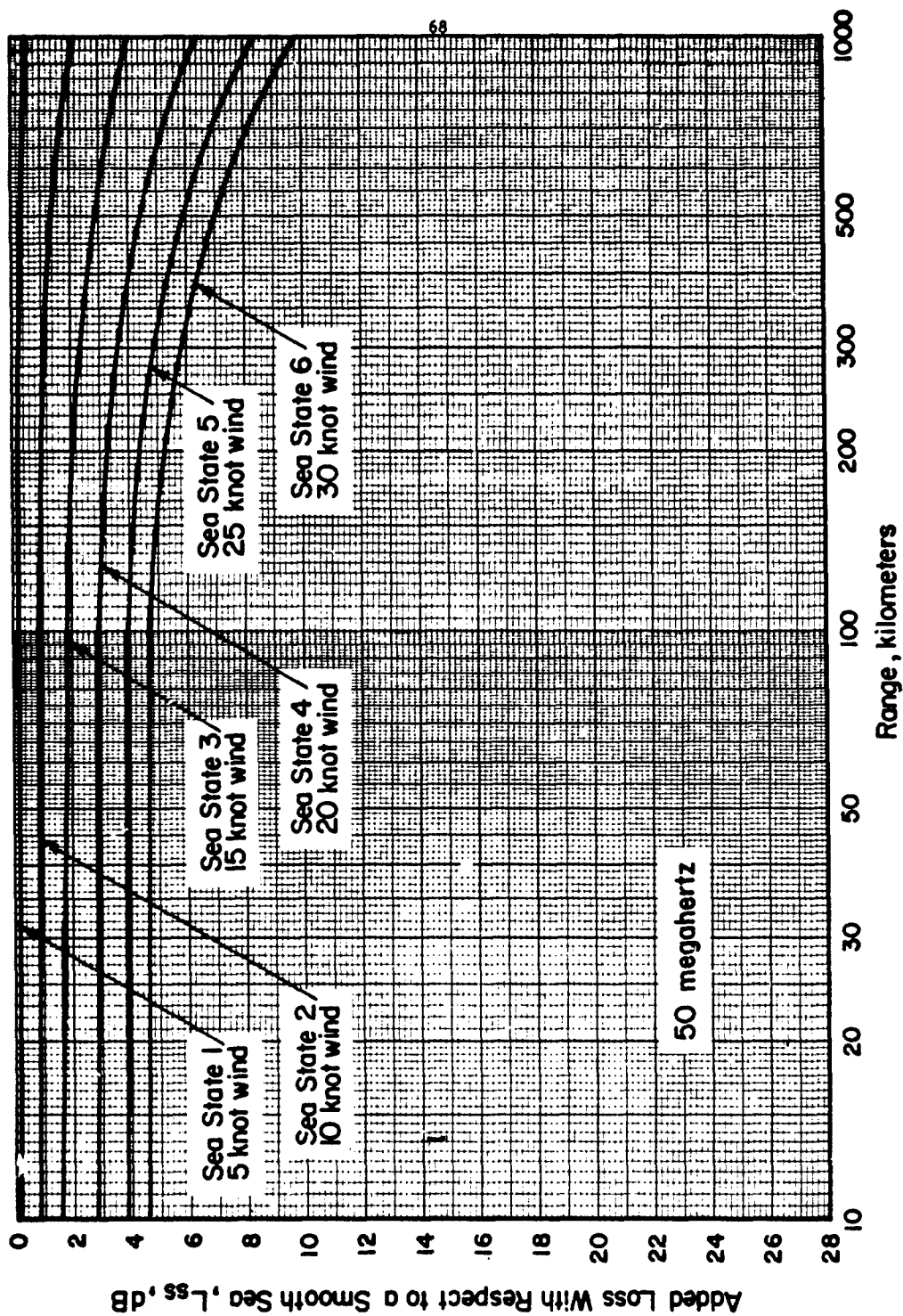


FIGURE 24. Added Transmission Loss Due to Sea State at 50 Mhz. Antennas are Located Just Above Surface. Neumann-Pierson Ocean-Wave Spectrum with Propagation in Crosswind Direction.

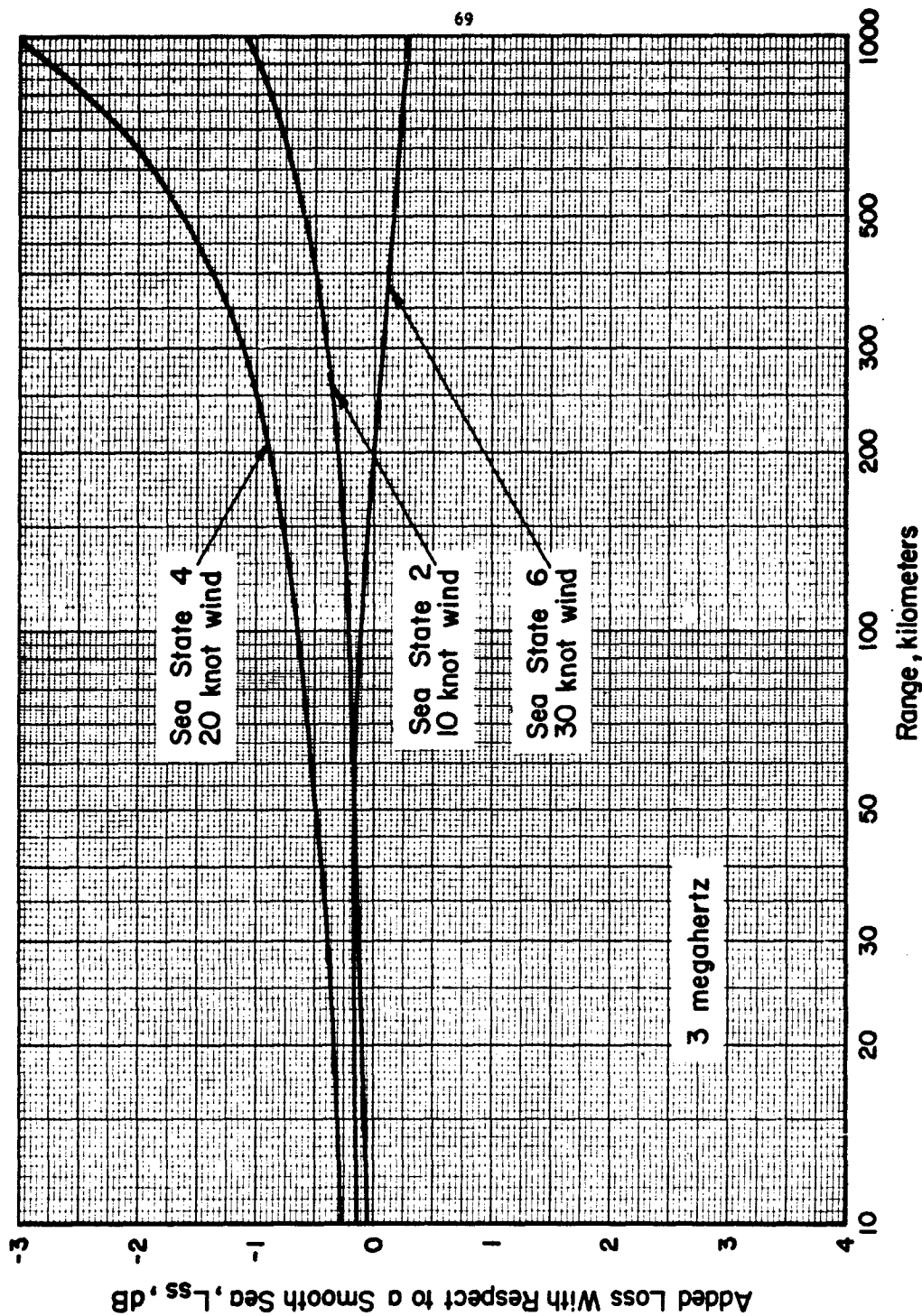


Figure 25. Added Transmission Loss Due to Sea State at 3 Mhz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.



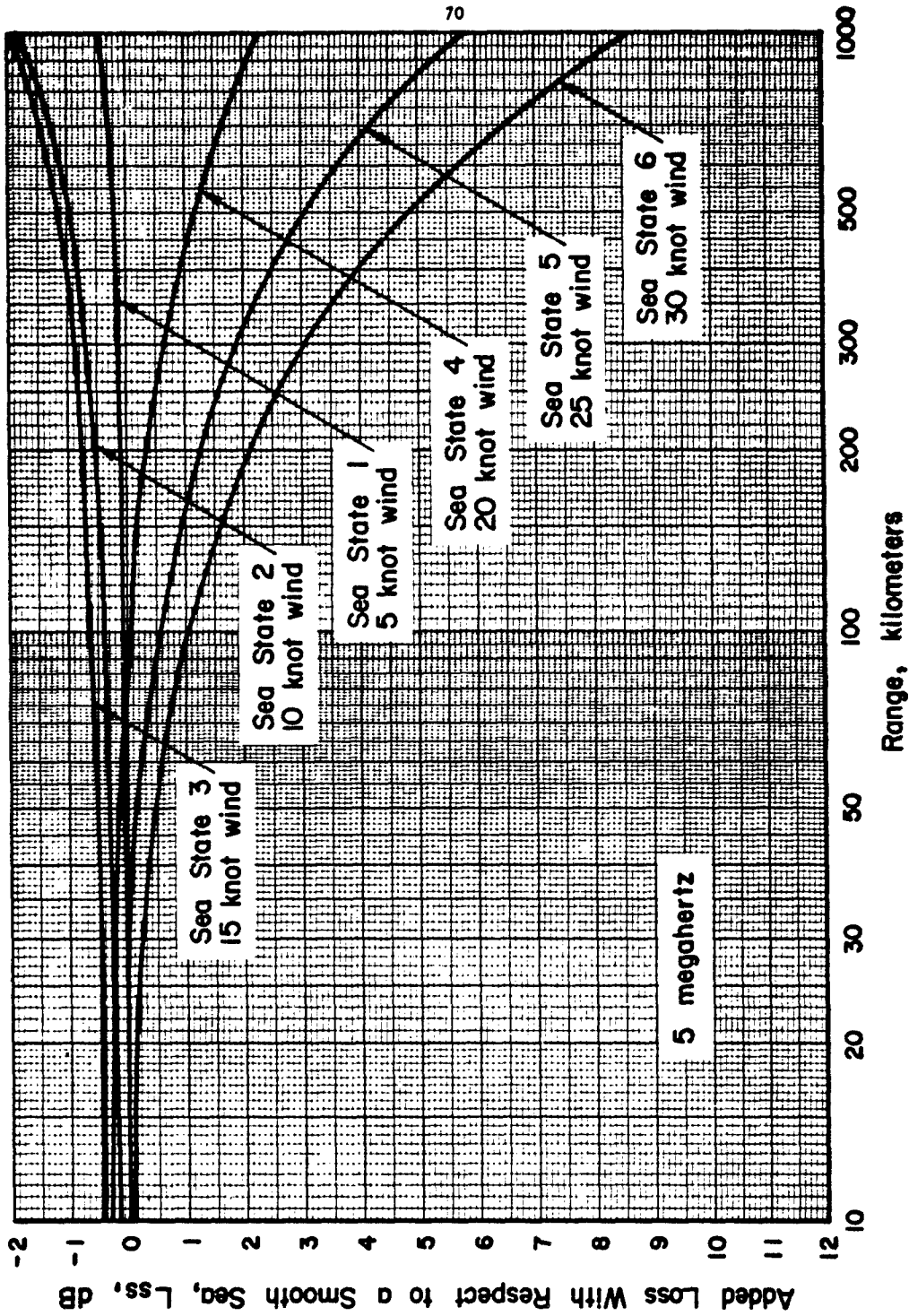


Figure 26. Added Transmission Loss Due to Sea State at 5 Mhz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

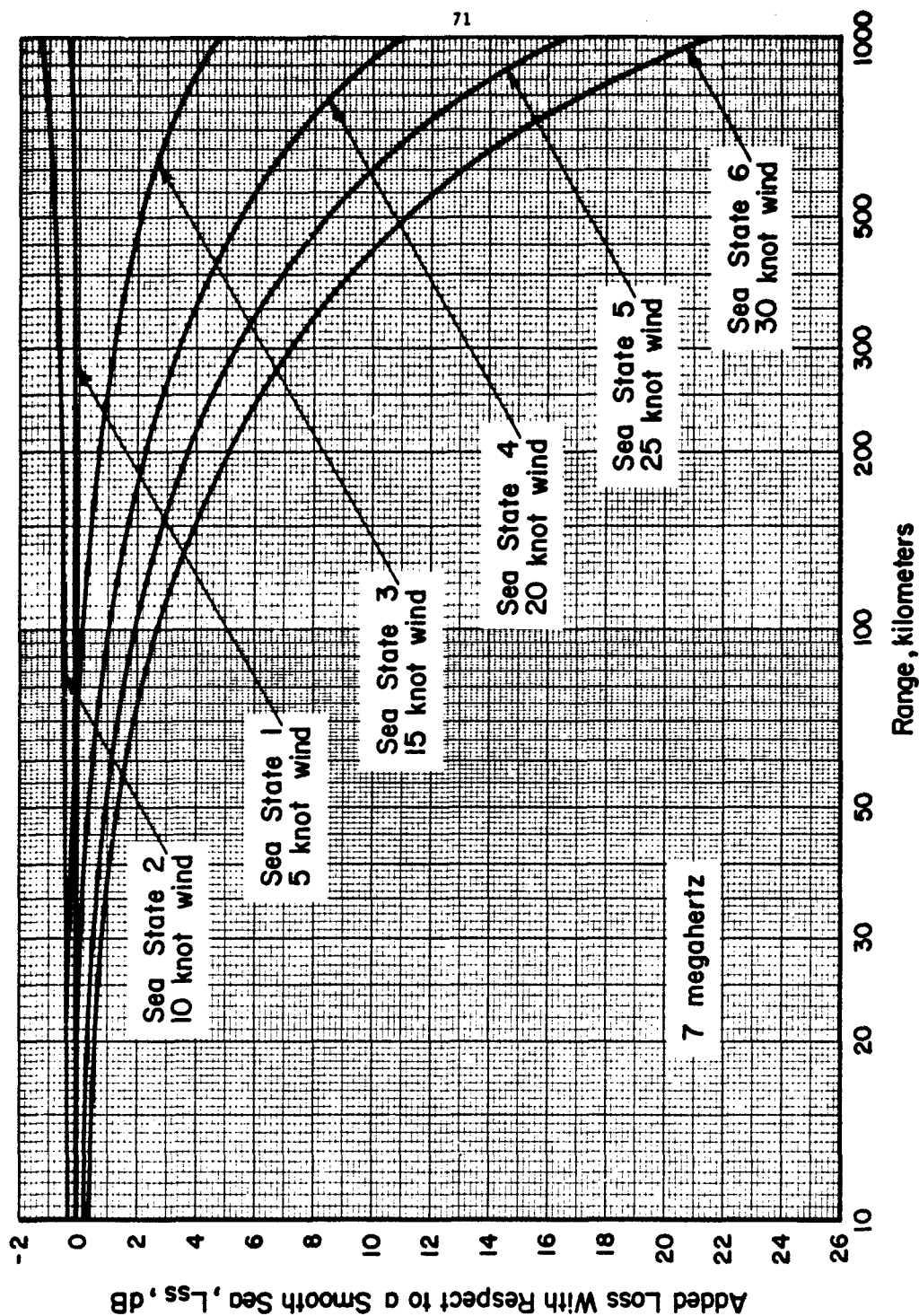


Figure 27. Added Transmission Loss Due to Sea State at 7 MHz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

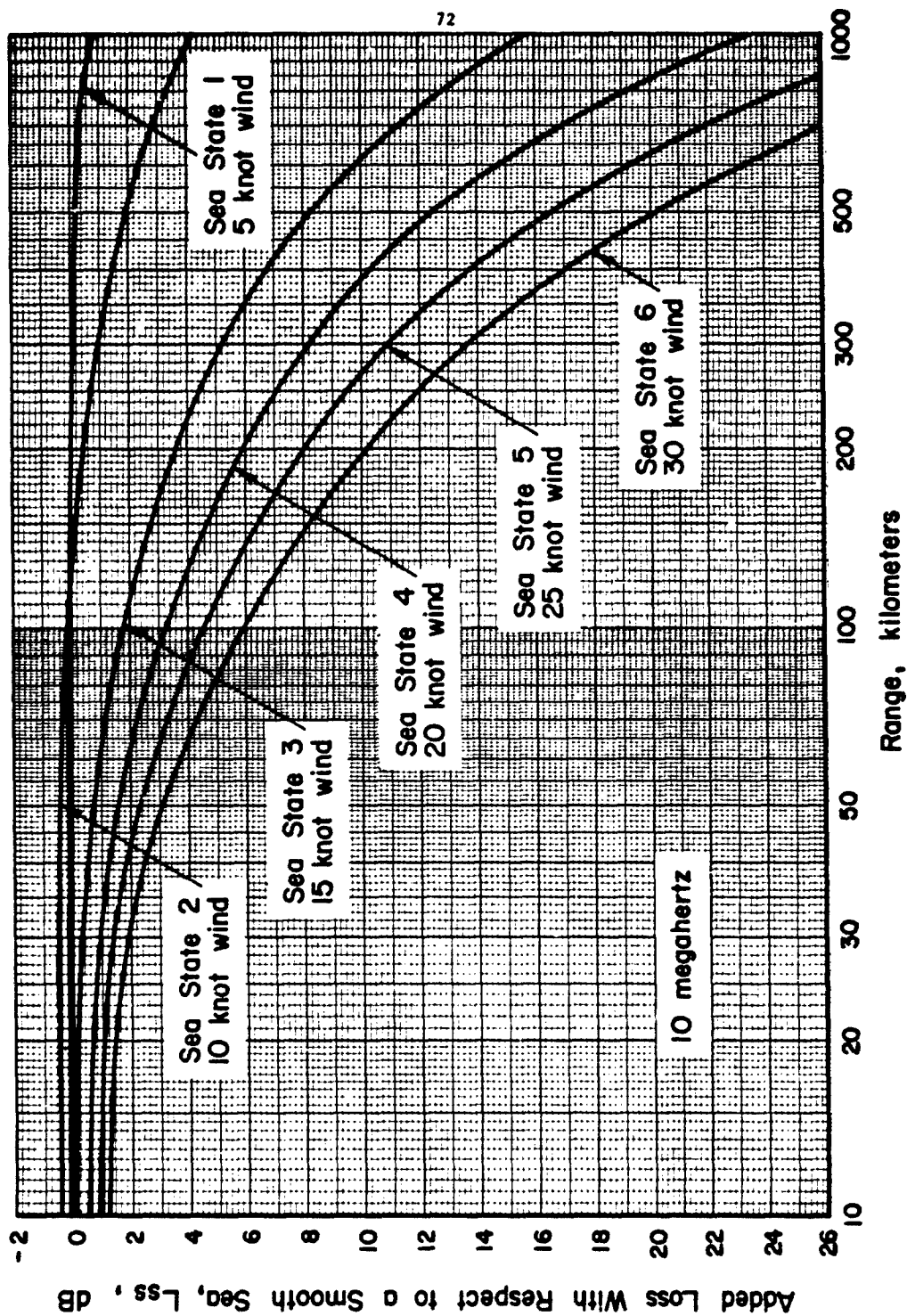


Figure 28. Added Transmission Loss Due to Sea State at 10 Mhz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

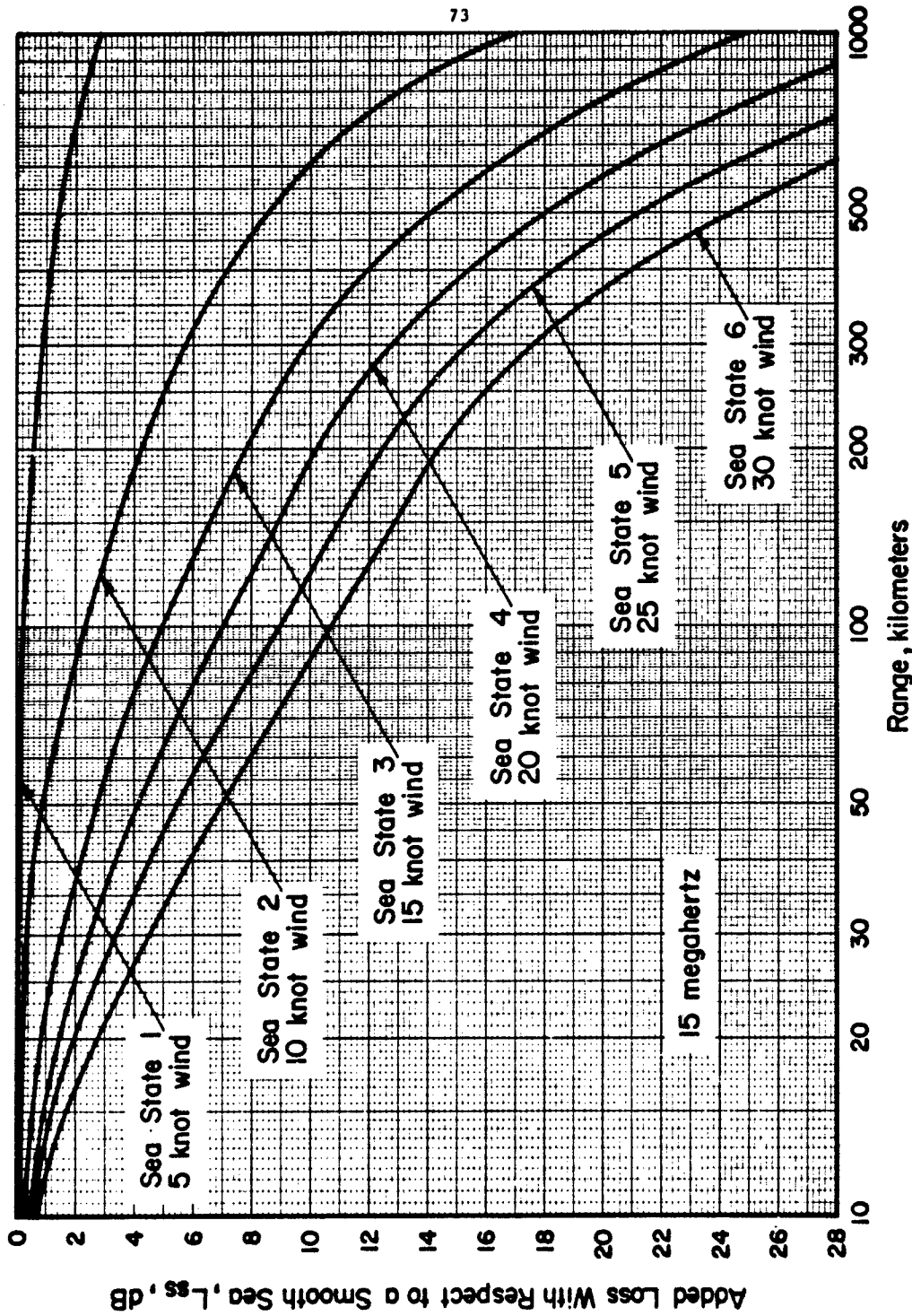


Figure 29. Added Transmission Loss Due to Sea State at 15 MHz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

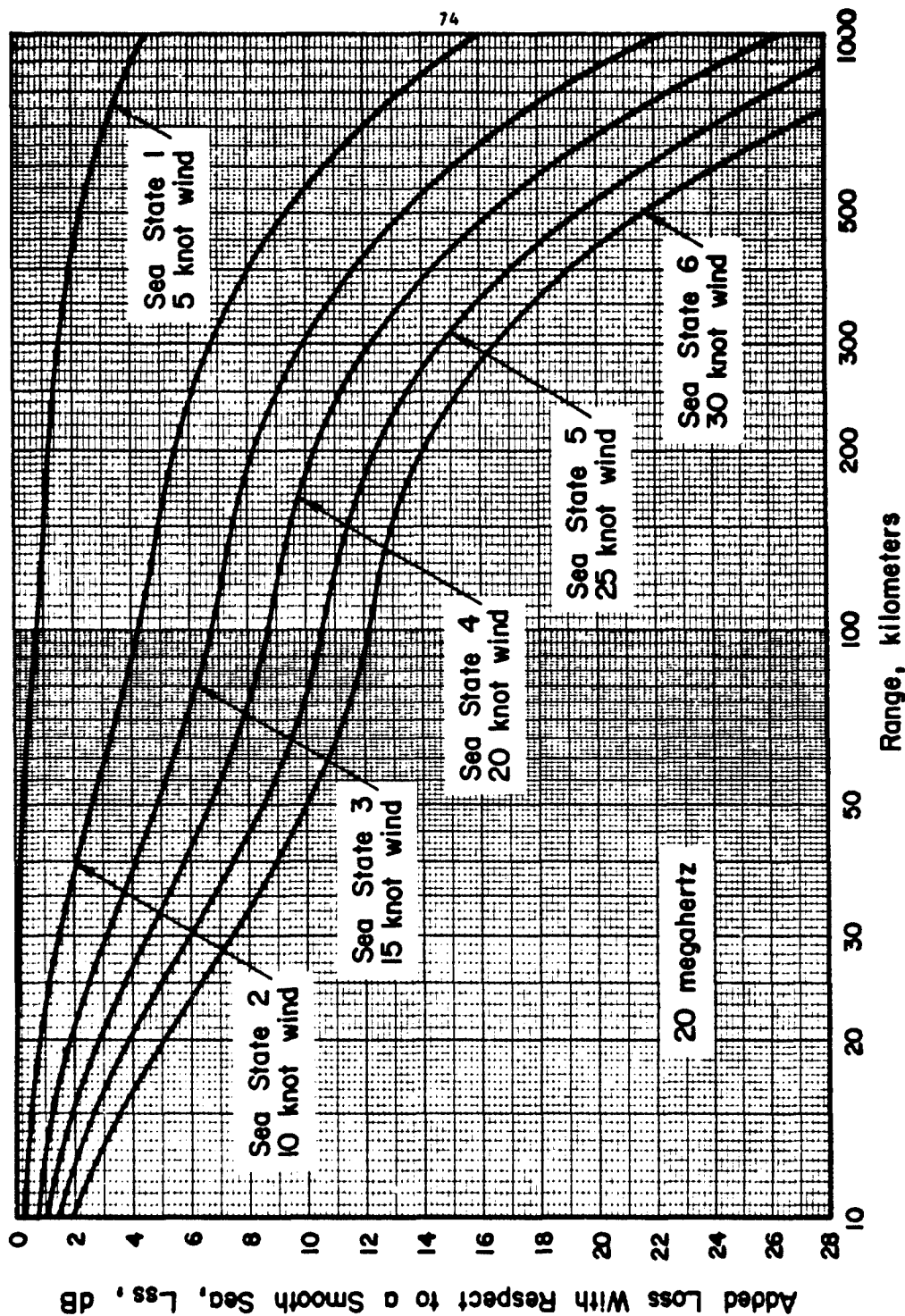


Figure 30. Added Transmission Loss Due to Sea State at 20 Mhz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

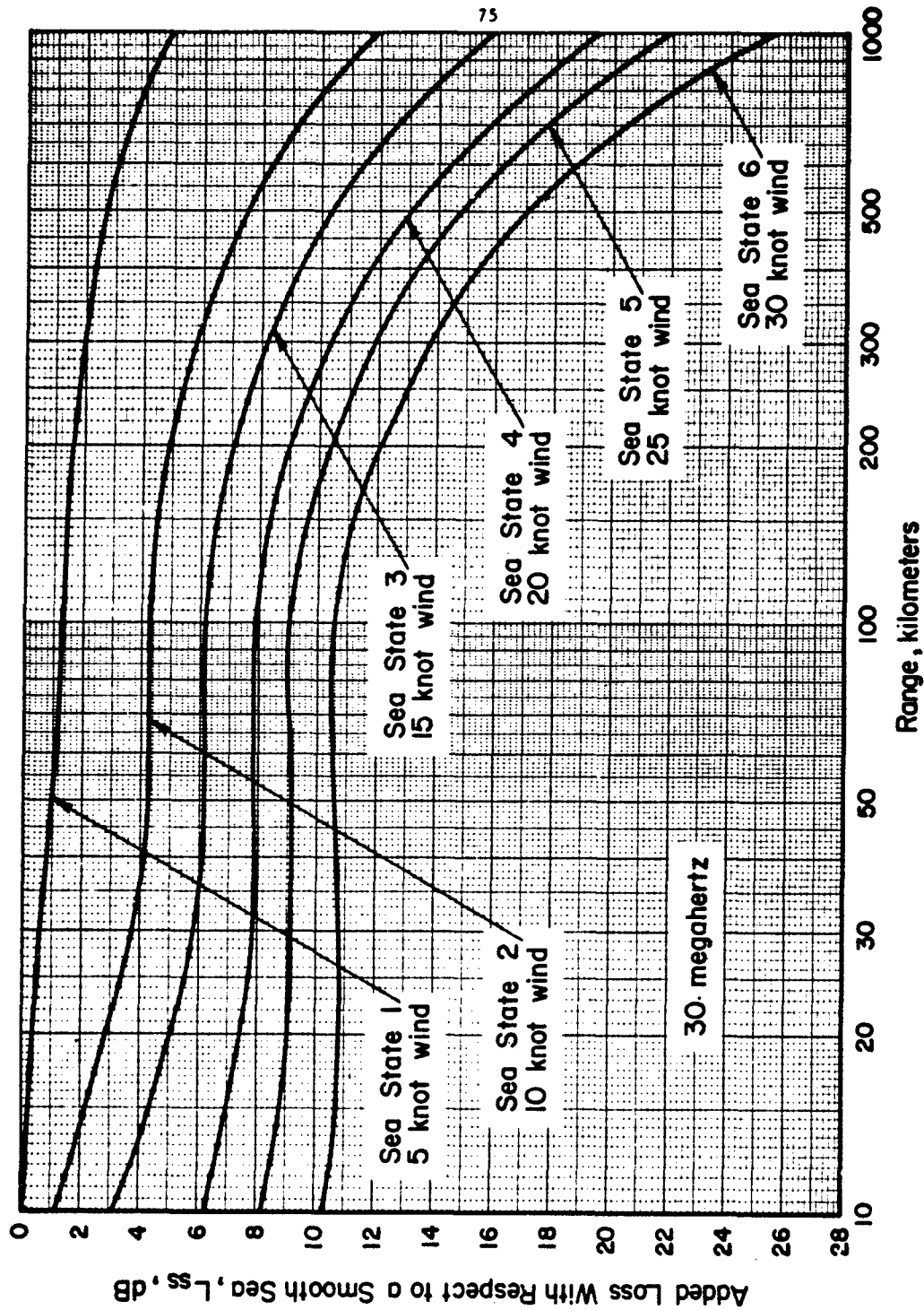


Figure 31. Added Transmission Loss Due to Sea State at 30 Mhz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.

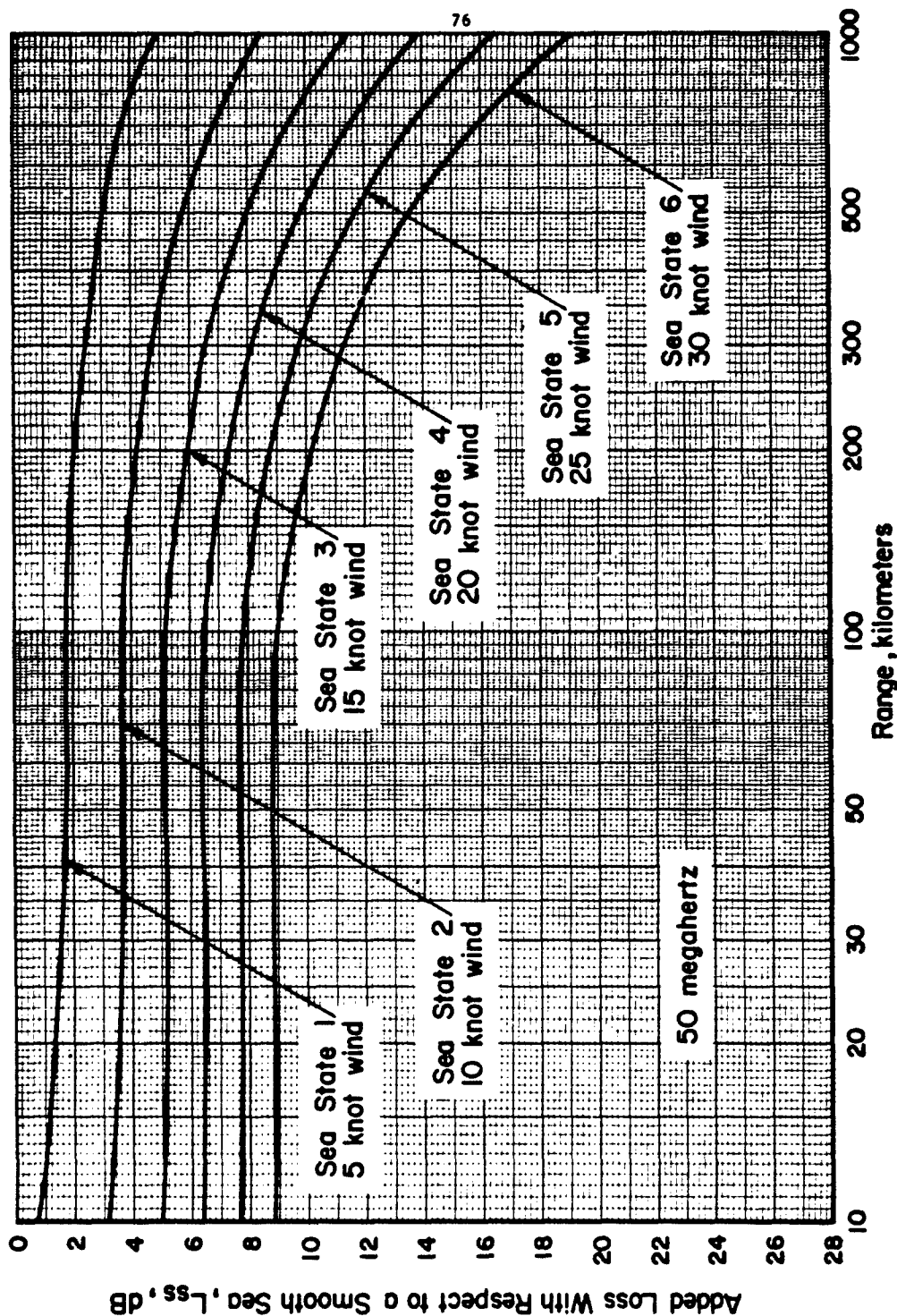


Figure 32. Added Transmission Loss Due to Sea State at 50 MHz. Antennas are Located Just Above Surface. Phillips Isotropic Ocean-Wave Spectrum.



Figure 33. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 1 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 3 (25 knot wind) Using Phillips Ocean-Wave Spectrum.



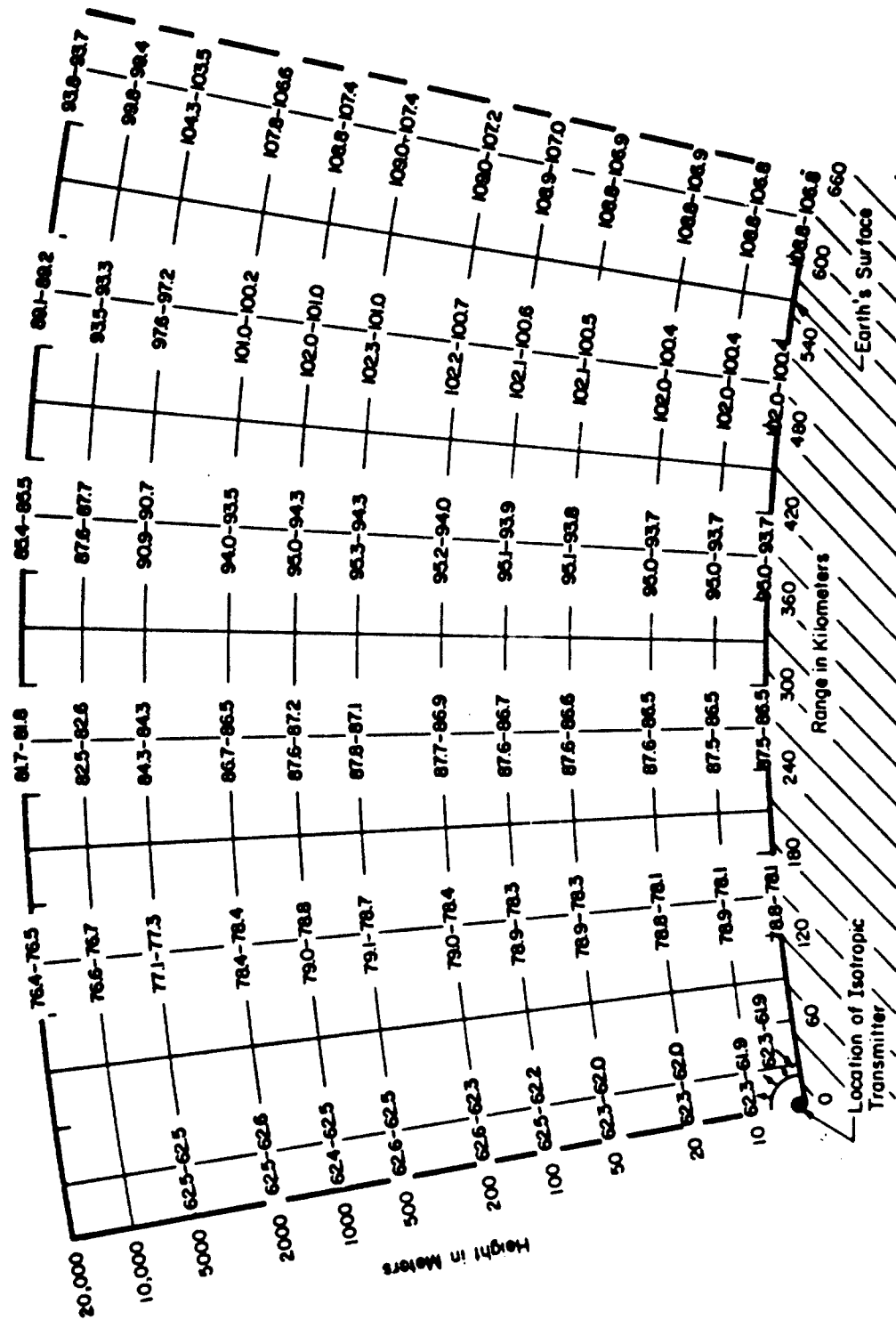


Figure 34. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 2 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

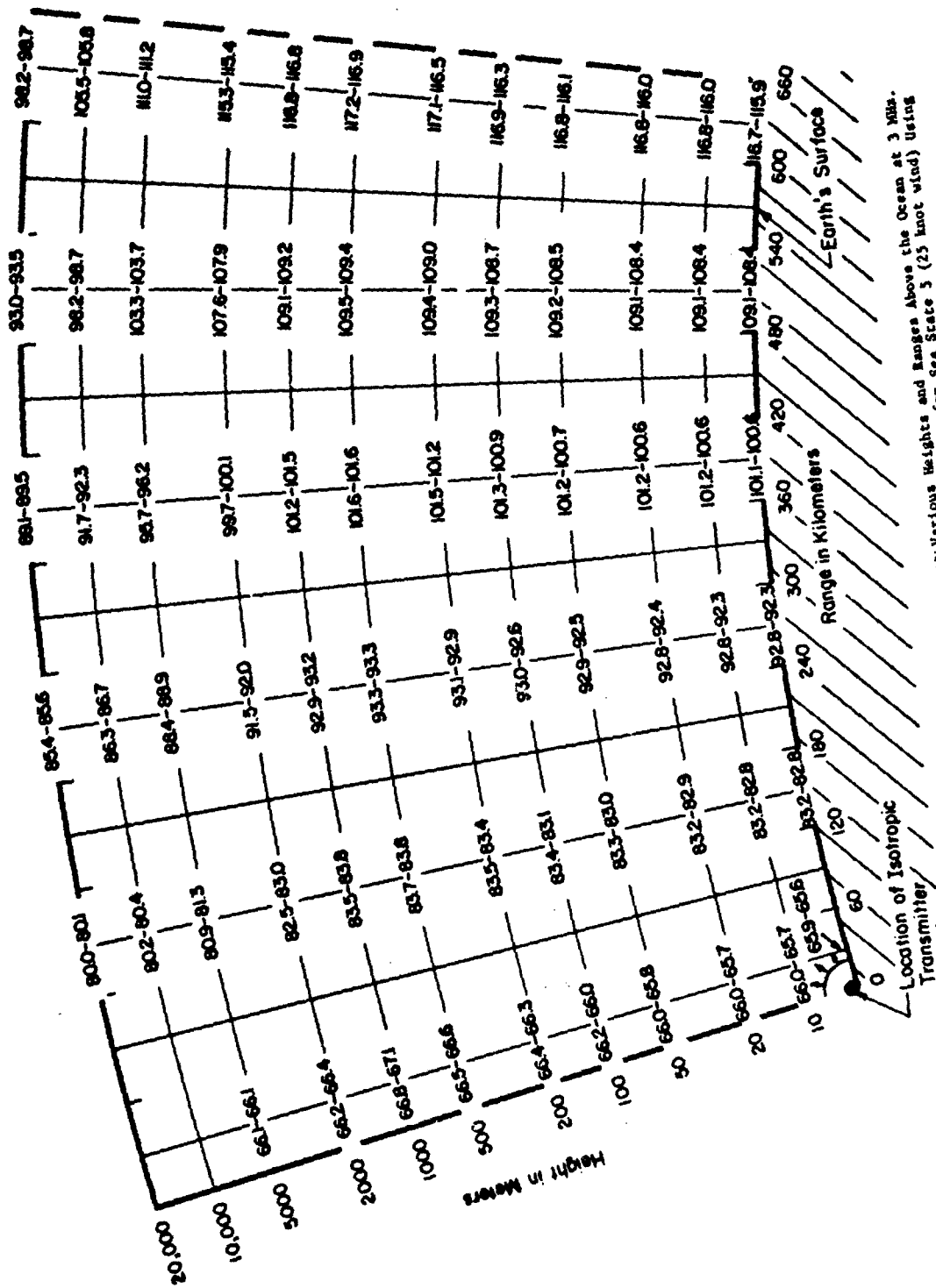


Figure 35. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 3 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 3 (25 knot wind) using Phillips Ocean-Wave Spectrum.

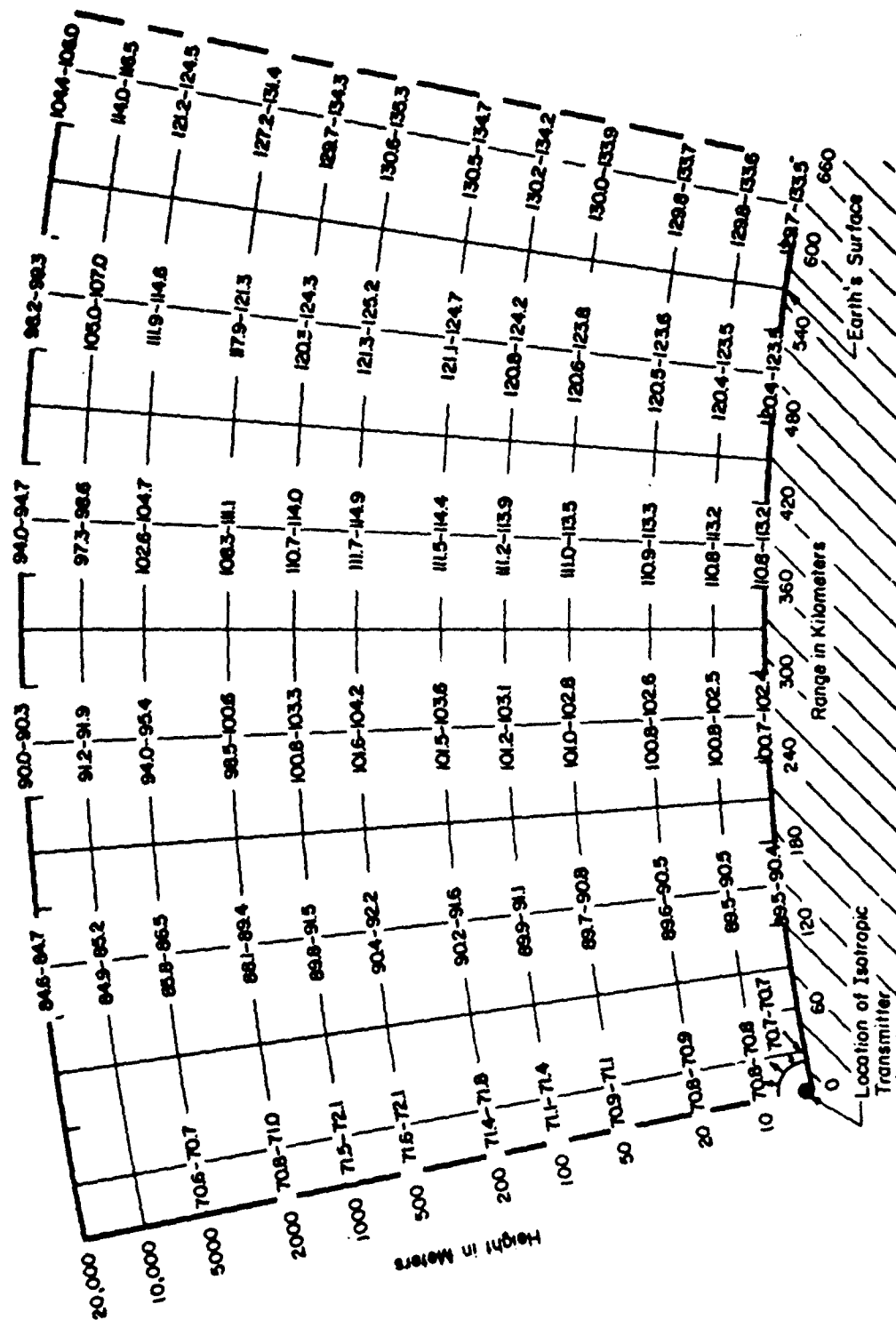


Figure 36. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 5 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

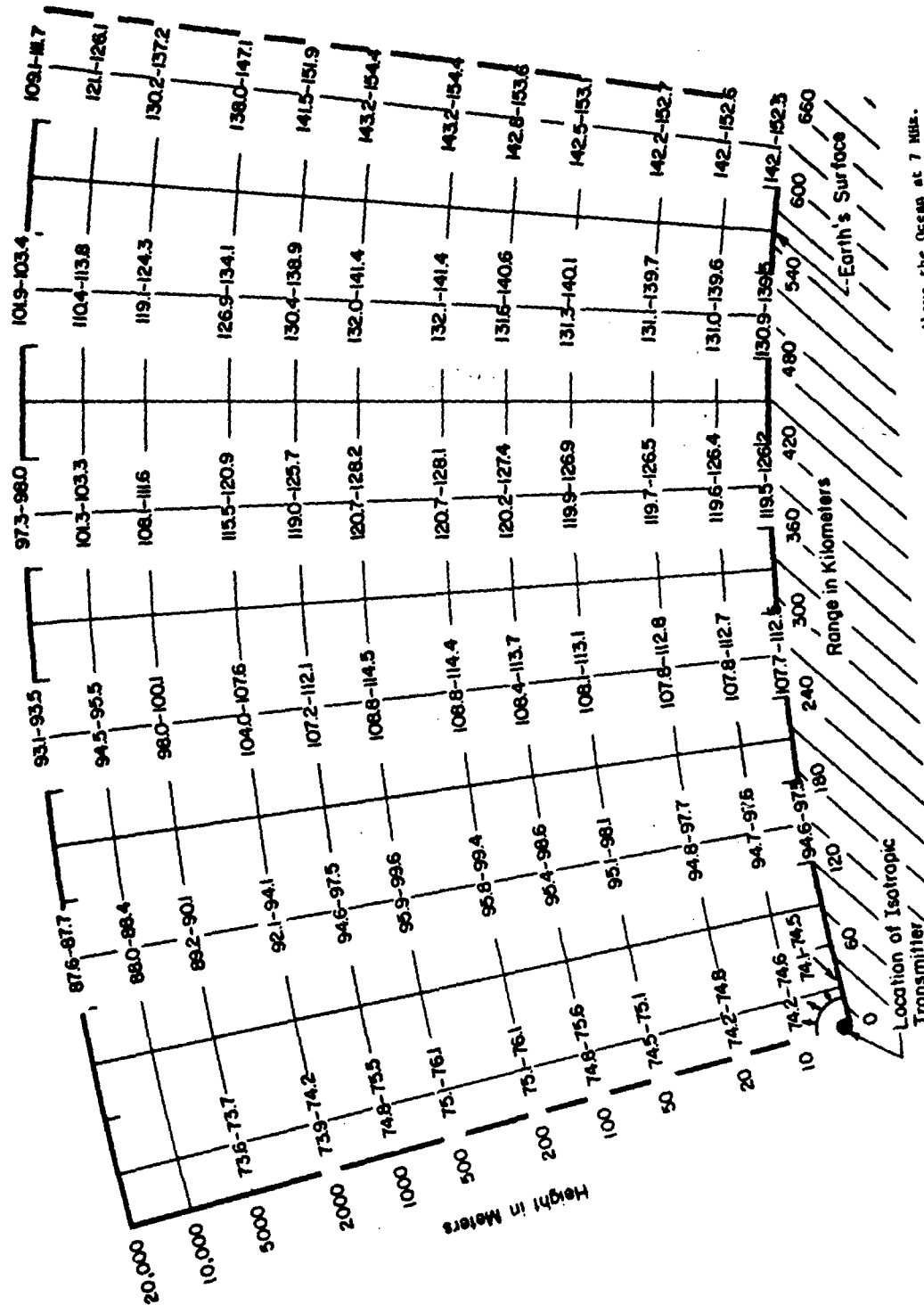


Figure 31. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 7 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

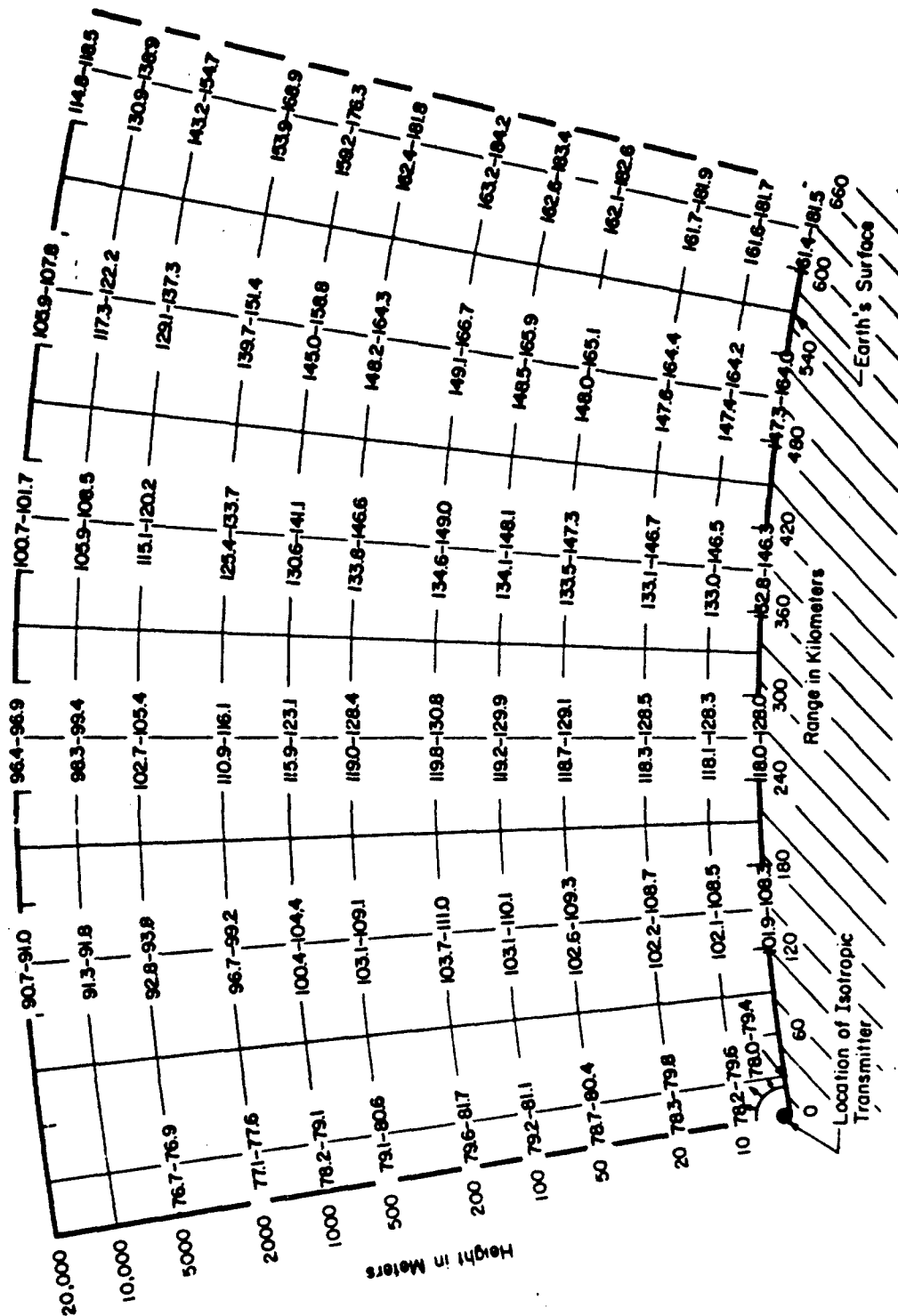


Figure 38. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 10 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

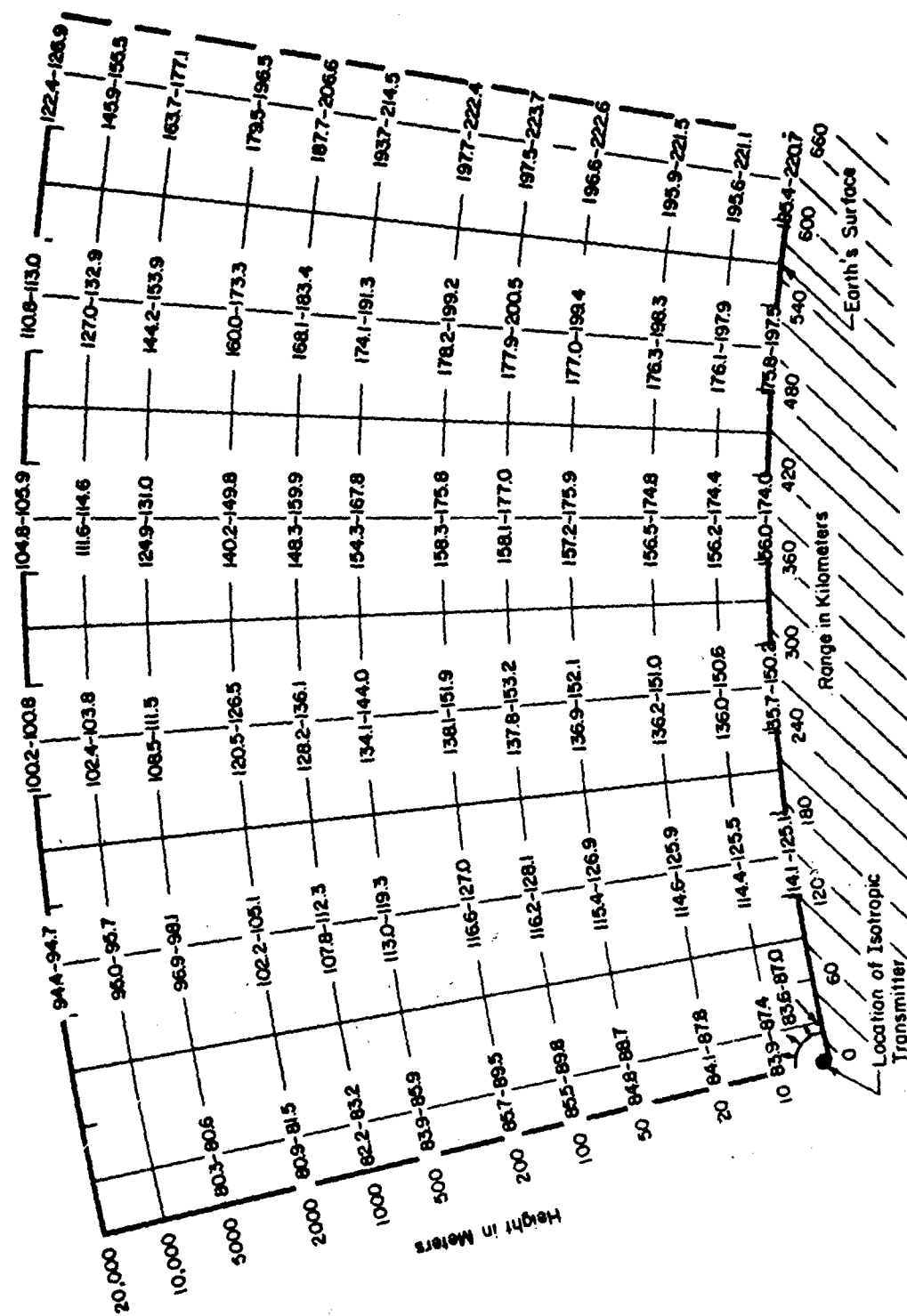


Figure 39. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 15 Mhz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) using Phillips Ocean-Wave Spectrum.

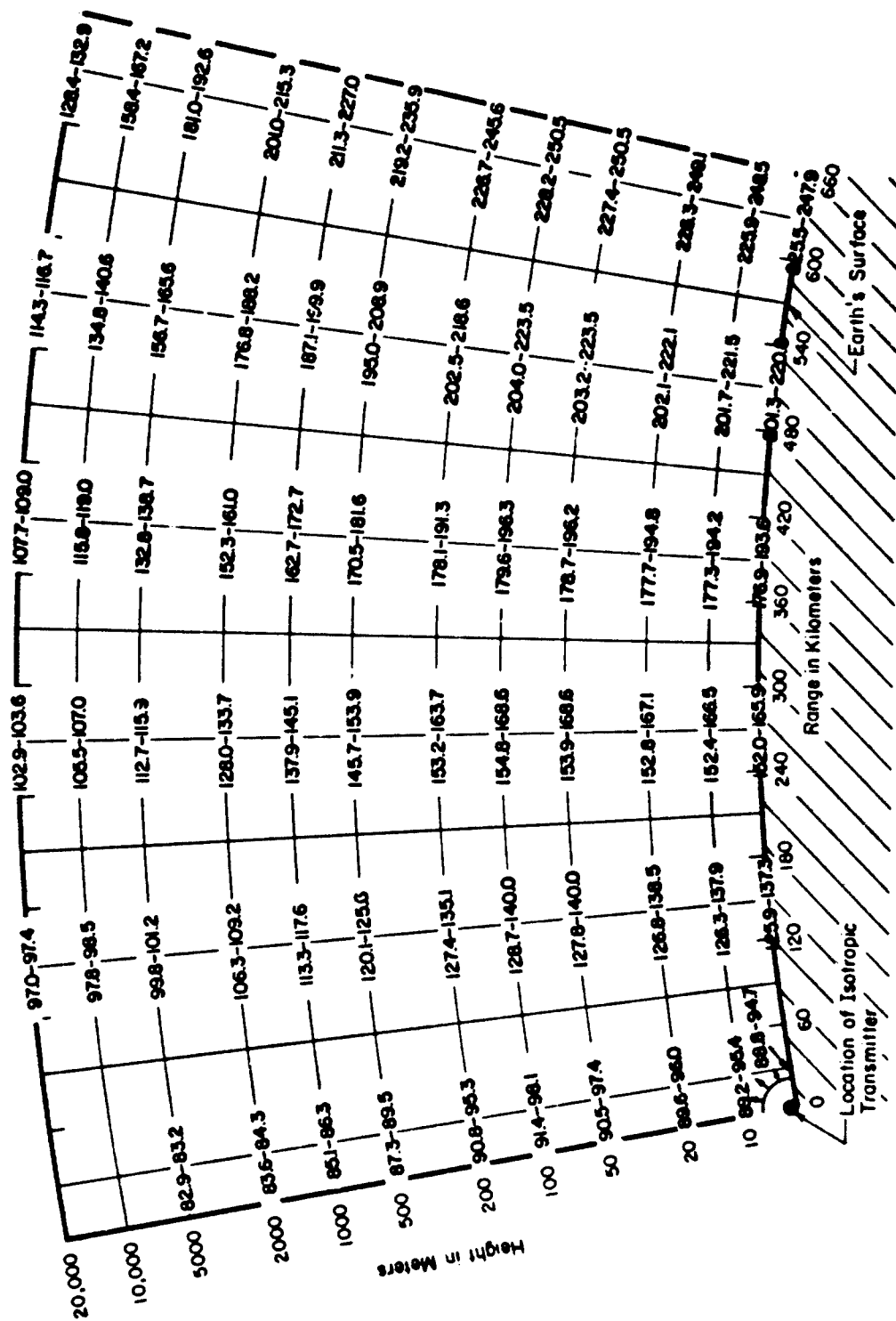


Figure 40. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 20 Mhz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

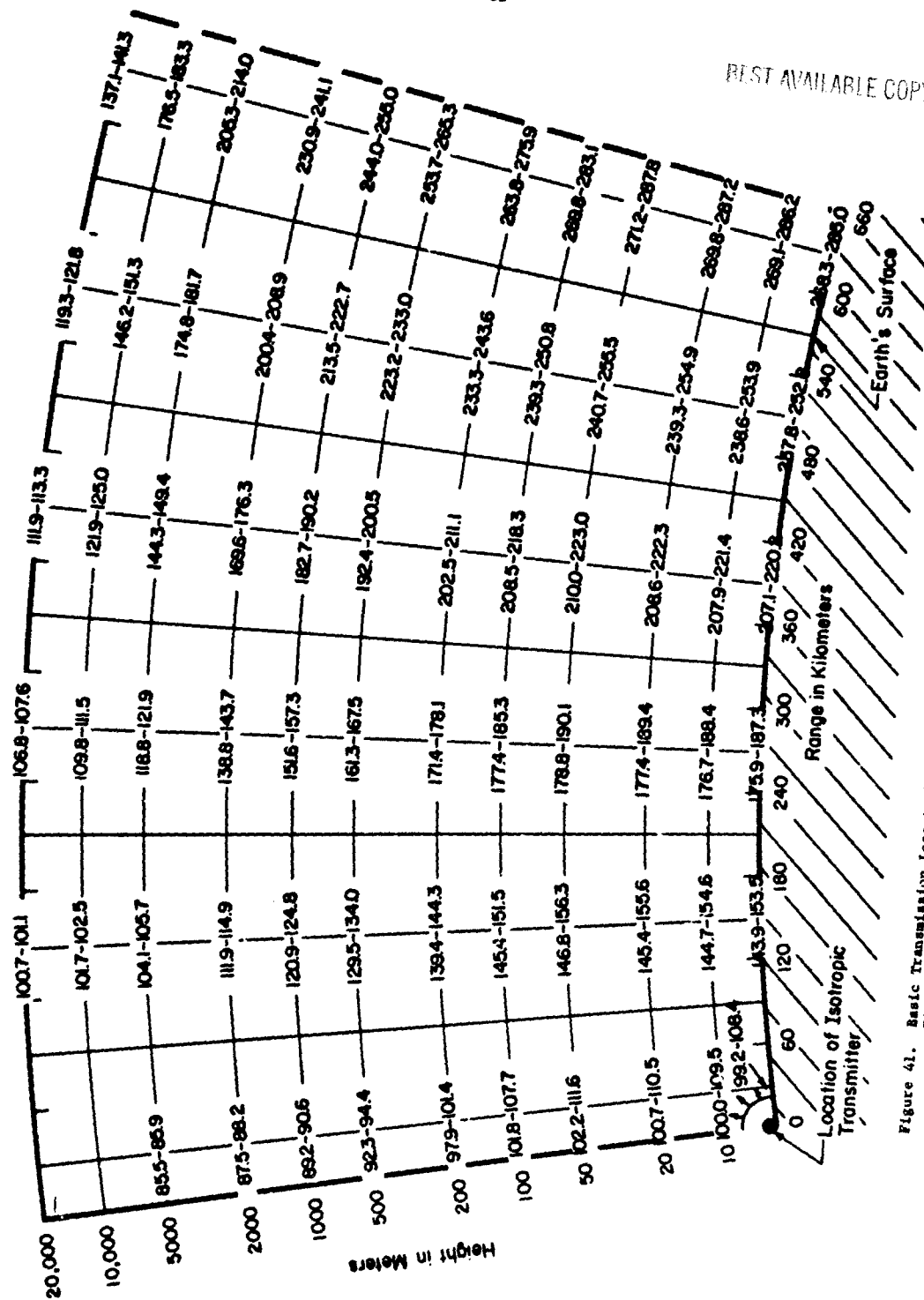


Figure 41. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 30 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 5 (25 knot wind) Using Phillips Ocean-Wave Spectrum.



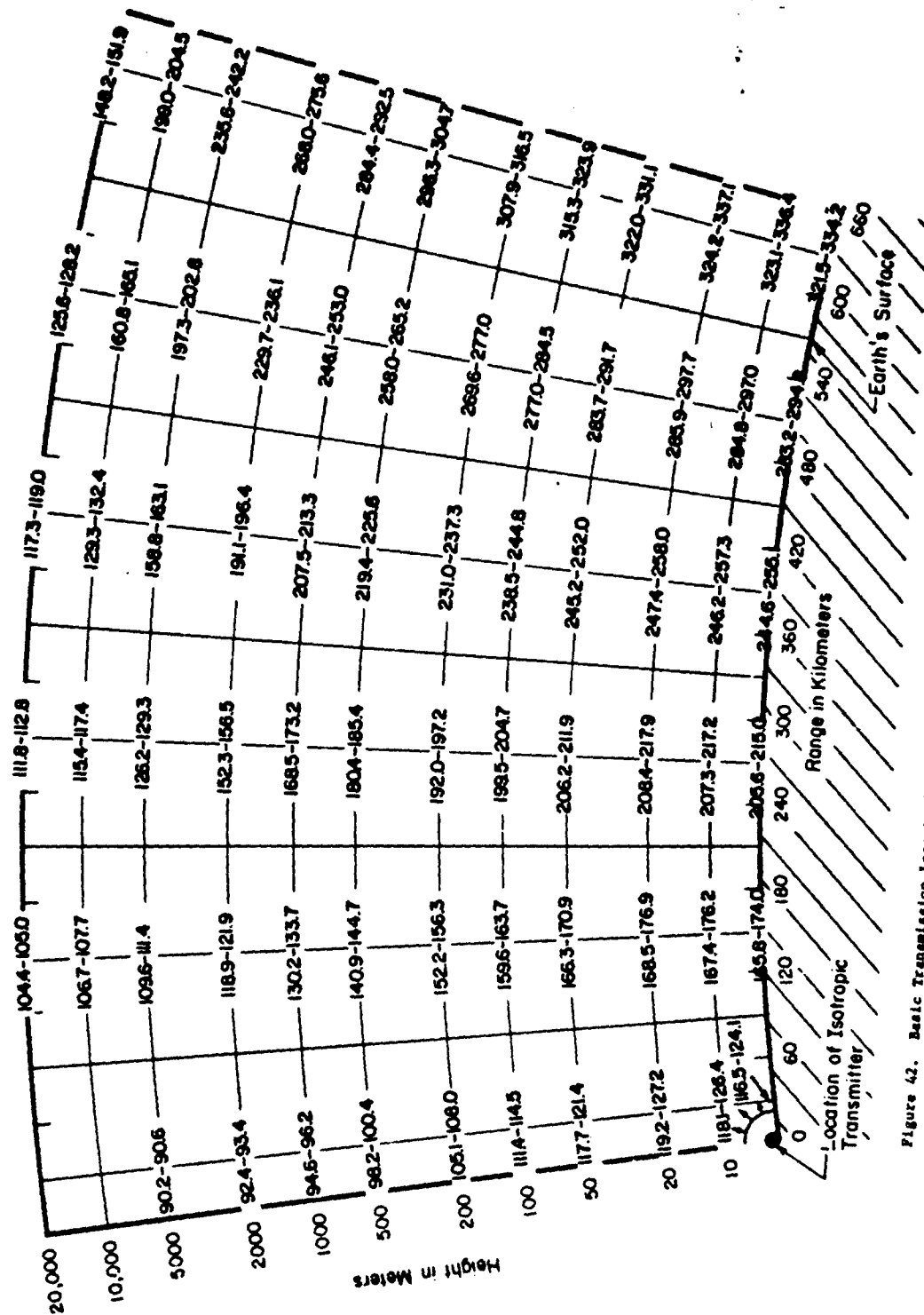


Figure 42. Basic Transmission Loss to Points at Various Heights and Ranges Above the Ocean at 50 MHz. First Number is for Perfectly Smooth Sea, Second is for Sea State 3 (25 knot wind) Using Phillips Ocean-Wave Spectrum.

The total loss is given as

$$10 \log_{10} \frac{P_t}{P_r} = L_b - G_t - G_r \quad (47)$$

From Figure 8, we find that at 300 km,  $L_b = 121.8$  dB for a smooth sea; using Figure 28 we note that this increases to 132.7 dB for a sea driven by a 25-knot wind. Hence,  $L_b - G_t - G_r = 98.8$  dB and 109.7 dB for these two sea conditions. Thus, for  $P_t = 1$  kW (average),  $P_r \approx 1.3 \times 10^{-7}$  Watts (average) when the sea is calm, but drops to  $P_r \approx 1.1 \times 10^{-8}$  Watts (average) in rough seas produced by a 25-knot wind.

#### B. Surface-to-Air Communication Problem

Here let us consider a ship-to-air communication application. The frequency is 20 MHz. We wish to determine the signal power received by an aircraft at 500 meters altitude (about 1500 feet) and 270 km away from the ship during calm sea conditions for 500 watts transmitted power. Furthermore, we wish to estimate how much higher the plane must fly at that range to receive the same signal power when the sea is aroused by a 25 knot wind. Possible ionospheric reflections are not to be included in the signal estimates. The aircraft free-space antenna gain is about 6 dB.

For the smooth-sea problem, we employ Figure 40 to find the basic transmission between two isotropic sources, one located on the surface and the other at 500 meters altitude and 270 km range. This is given by the first number at the appropriate grid point on the chart, i.e.,  $L_b = 145.7$  dB. Then using Equation (47), we find that the received power will be  $P_r \approx 1.35 \times 10^{-11}$  watts.

At that same altitude, we see from the second number at the same grid point that the transmission loss increases to 153.9 dB when the sea is fully aroused by a 25 knot wind. This 8.2 dB drop in received signal, however, can be recovered by climbing in altitude to the next grid point (i.e., 1000 meters or 3000 feet), at which the loss drops back to 145.1 dB, near the original 145.7 dB figure for the smooth sea.

## VI. SUMMARY

This report has had one principal purpose throughout: to derive some estimates of the effects of sea state on ground-wave propagation at HF and VHF. At the outset of the analysis it was established that the only truly applicable boundary condition is the Leontovich, or impedance boundary condition. This boundary condition, when applied to the sea, correctly accounts for the slight losses associated with the finite water conductivity. With this boundary condition, the derivation followed the classical statistical boundary perturbation approach developed by Rice. The effective surface impedance accounting for the roughness was derived in this manner for a vertically polarized wave which appears to propagate locally over the surface at the pseudo-Brewster angle. Since the ground wave propagates locally at this angle, the effective surface impedance thus derived can be used in any of the conventional treatments of radiation and propagation over a spherical earth with an impedance boundary.

The effective surface impedance obtained here is seen to consist of two terms: one which is the impedance of a otherwise smooth planar surface of sea water, and the second which is due to the roughness height spectrum of the ocean waves. This latter term preserves the conductivity dependence, and the error involved in assuming a perfectly conducting surface can thus be clearly seen. The increase in the surface impedance due to the roughness is due largely to two phenomena: the resistance increase is caused by removal of energy by Bragg scatter from the ground wave. The inductive reactance increase is due to the shorter roughness wavelengths which do not scatter, but produce evanescent modes near the surface. The latter phenomenon is identical to that behind the use of short-period corrugations to "trap" energy near guiding, slow-wave structures.

The analysis presented has emphasized the electromagnetic derivation. No attempt was made to analyze or improve the oceanographic models for the sea height spectrum. In order to obtain estimates of sea state effects, two commonly seen ocean-wave spectra models were used: the Neumann-Pierson and the Phillips spectrum. The effective surface impedance based on these two models was numerically calculated for various frequencies and wind conditions. Finally, we used these surface impedances in a standard program to compute the basic transmission loss across the sea vs range

for various frequencies and winds. Of the two ocean-wave models, it is believed that the Phillips spectrum is somewhat more justified than the Neumann-Pierson directional model.

To illustrate how these curves can be applied to communication problems we traced through some sample problems. The curves can also be applied to radar applications in a straightforward manner.

#### VII. CONCLUSIONS

Wind speed and the resulting ocean roughness will definitely affect the propagation of a ground wave in the HF/VHF regions. This roughness effect is generally negligible below a frequency of 3MHz. At 15 MHz and a range of 100 miles, the received signal over the Atlantic is shown to vary as much as 15 dB due to sea state. Above about 50 MHz and for wind speeds greater than 25 MHz, the techniques derived here are no longer valid because of the assumptions inherent in the analysis.

Any deviations of measured propagation loss from that predicted by our curves are not believed due to any shortcomings of the electromagnetic analysis. They are due rather to the empirical ocean-wave spectra models used here. Reliable directional spectra, as required by our equations, have not been measured in sufficient detail by oceanographers to permit the establishment of an empirical model. In addition, the only isotropic spectra models available are based on wind-driven seas and do not include the effects of swell generated by storms in other areas. There is at present no reasonable estimate of the error resulting from this neglect of swell.

Measurements of ground-wave losses over the sea are in progress and results should be available for comparison in the near future. As these measurements are being made, Raytheon is also recording the sea conditions for correlation with the signal strength. Along with their measurements of radar sea clutter, this data could, as a by-product, provide welcome information about the ocean-wave spectrum.

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APPENDIXGround-Wave Bibliography

The subject of ground-wave propagation across the earth is one of the most widely treated technical subjects of this century. We attempt to list here most of the principal open-literature publications relating to this subject. We apologize for omissions, of which there are certainly many. The list here has invariably failed to include many reports, even of an unclassified nature, as well as many foreign articles on the subject.

The bibliography does not include treatments of the effects of the ionosphere or atmosphere. Nor is ground scatter (clutter) considered.

So as to show the historical evolution of the subject, we break the bibliography into decades. All of the publications generated within a given decade are then arranged alphabetically by author.

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